

Final Exam

ENGR 12, Spring 2026.

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Exam Date	Sun, May 10, 2026
Duration	Three hours (180 minutes)

Name

Instructions

Answer all questions. Partial credit is available for all partial answers. Some problems may require less work than others, but they are all worth the same number of points. You are allowed to use a scientific calculator. No external resources, other than the accompanying appendix, may be consulted. Pages 3, 8, 9, 11, 13, 15, 18, 19 and 20 are blank. Please do not separate sheets, but if you do, write your name on the separated sheets and place them in order.

1 Block Diagrams

Consider the block diagram below.

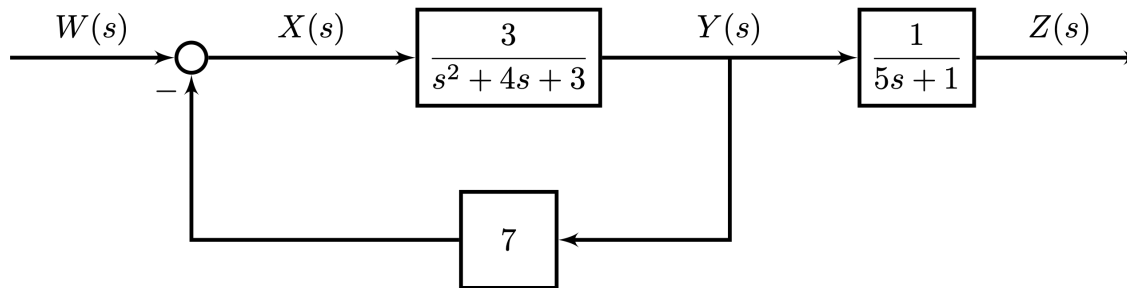


Figure 1

1.1 Identifying Transfer Functions by inspection

1. Write down the transfer function between $Z(s)$ and $Y(s)$.
2. Write down the transfer function between $Y(s)$ and $X(s)$.

1.2 Calculating Transfer Functions

Calculate the transfer function that relates the input $W(s)$ to the output $Y(s)$ and give your answer as a rational function (i.e., a ratio of two polynomials) in s .

1.3 Relating $Z(s)$ and $W(s)$

Write down an equation, in terms of s , that relates $W(s)$ and $Z(s)$. Your equation should not contain $X(s)$ or $Y(s)$.

i Note

Your equation need not be simplified or put into any particular form. Any correct equation relating Z to W in terms of s will suffice.

1.4 The Order of a system

What is the order of the linear system shown by Figure 1 considered as a whole?

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2 Draw Bode Plot

Make a qualitatively accurate sketch of the Bode plot for a system whose transfer function is

$$T(s) = \frac{s/1000 + 1}{10s + 1}$$

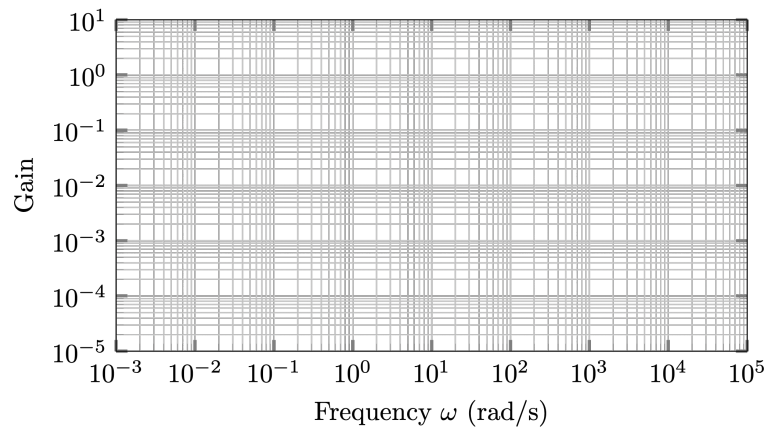


Figure 2

3 Stability

Write down either the differential equation or the transfer function for a second-order system that is unstable. Briefly show how you know that this system is unstable.

4 Free Response of Second-order systems

Four second-order linear systems, labeled A, B, C and D, are to be compared regarding their free response when initialized with identical initial conditions. For each system, the transfer function has exactly two complex conjugate poles given by

$$s_{1,2} = r \pm i\omega_d,$$

where r is the real part of the pole and ω_d the imaginary part of the pole. The values of $s_{1,2}$ for each system are labeled on the following diagram, which shows the real and imaginary parts of the poles of each system on the horizontal and vertical axes respectively.

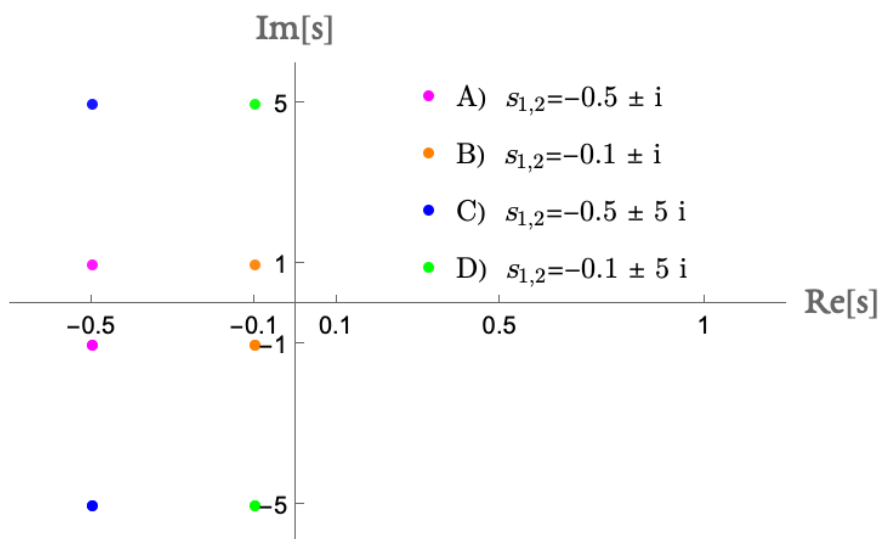


Figure 3: Poles of the transfer function for systems A through D, shown on the complex plane.

The free response of all four systems has been plotted against time in Figure 4, without labels showing which system corresponds to which graph. **Label each panel of Figure 4 with letters A through D.** There is only one label for each graph, and each graph has a label.

i Note

Correct labels will receive full credit. If one or more labels are incorrect, partial credit may be given for other work.

i Note

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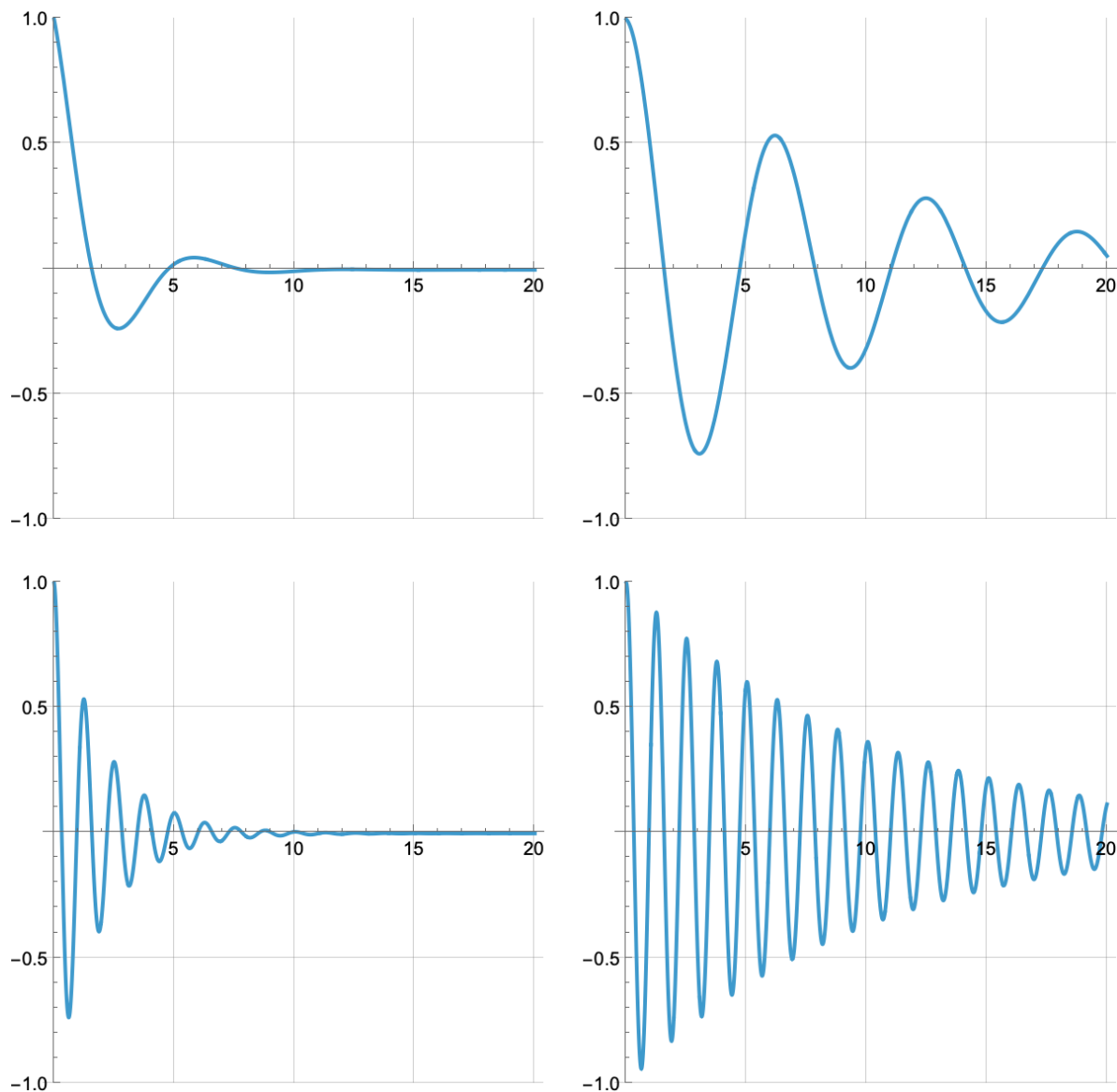


Figure 4: $x(t)$ for the free response of the four systems A, B, C and D. Label each panel with a letter.

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5 Spring-Mass Systems

Consider the system shown in Figure 5. A box of mass m is supported on frictionless rollers and is connected by a dashpot to the wall on the right and by a spring to a massless bar on the left, which also moves frictionlessly. The massless bar has position x_{in} and the mass has position x , such that when $x = x_{\text{in}} = 0$, there is no compression or extension in the spring.

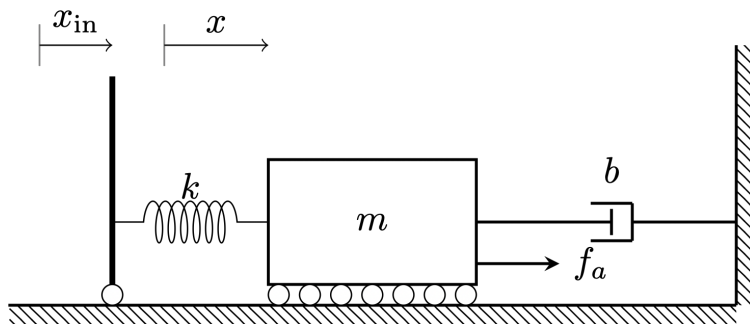


Figure 5

Considering the inputs to this system to be f_a and x_{in} , write down the state-space equations for this system using the form

$$\begin{aligned}\dot{\mathbf{x}} &= A\mathbf{x} + B\mathbf{u} \\ \mathbf{y} &= C\mathbf{x} + D\mathbf{u}\end{aligned}$$

where the inputs are f_a and x_{in} and the outputs are: (1) the total compressive force on mass m and (2) the distance between the massless bar and the mass m .

i Note

In your answer, you should fully specify the matrices A , B , C and D , as well as the vectors $\mathbf{x}$, $\mathbf{u}$ and $\mathbf{y}$.

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6 Impulse and Step Functions

Consider the circuit shown below.

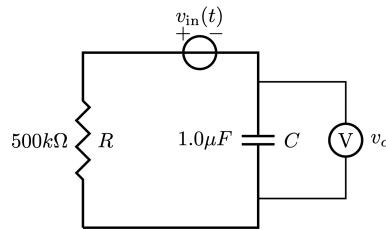


Figure 6

Initially, there is no energy stored in the capacitor. If the voltage source $v_{in}(t)$ is given by the sum of the following two functions:

1. A unit impulse input at time $t = 2$, and
2. A unit step input starting at time $t = 4$,

Sketch a quantitatively accurate graph of the voltage across the capacitor, v_c , as a function of time, on the following axes.

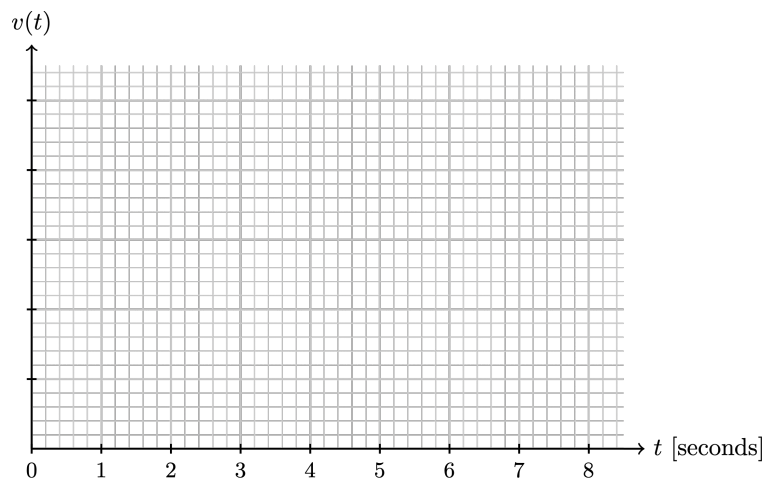
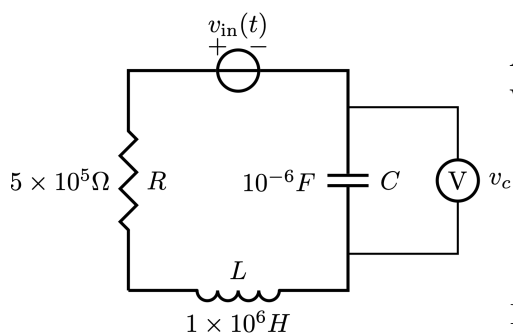


Figure 7

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7 'Pulse response'



An RLC Circuit, shown in Figure 8, has a variable input voltage v_{in} given by a 'pulse' function

$$v_{in}(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 \leq t < 20 \\ 0 & t \geq 20. \end{cases}$$

It is described by the equation $LC\ddot{v}_c + RC\dot{v}_c + v_c = v_{in}$

Figure 8

The voltage across the capacitor is initially 0.0 Volts.

On Figure 9, sketch a qualitatively accurate graph of the voltage across the capacitor, v_c , against time.

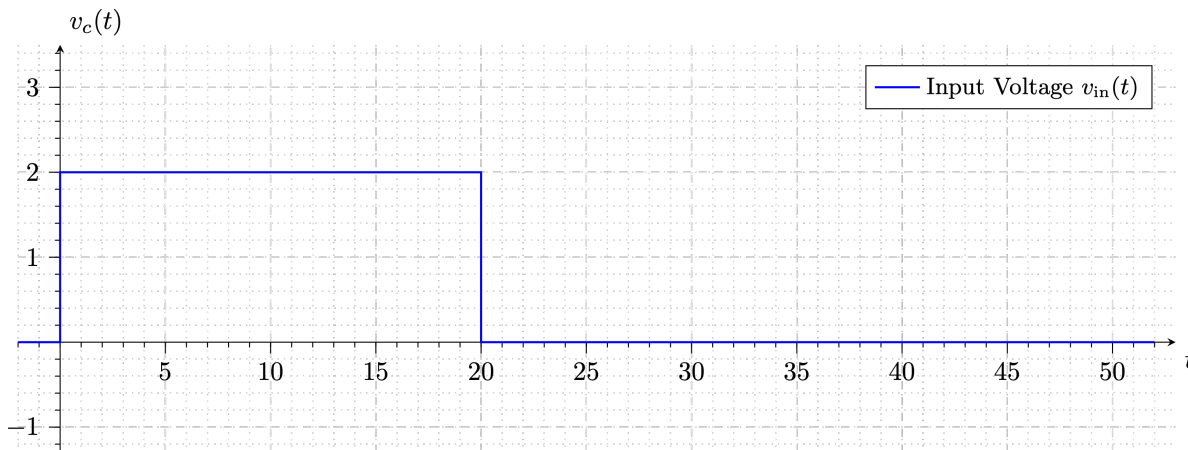


Figure 9: Note time is measured in seconds.

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8 The Fourier Transform and Bode plots

8.1 MATLAB's `fft` function

MATLAB's `fft` function, which stands for 'Fast Fourier Transform', can be used to extract the frequency content of a signal. In Problem Set 13, we saw that `fft` can be used to transform data from the time domain (panel (a) of Figure 10) to the frequency domain (panel (b) of Figure 10).

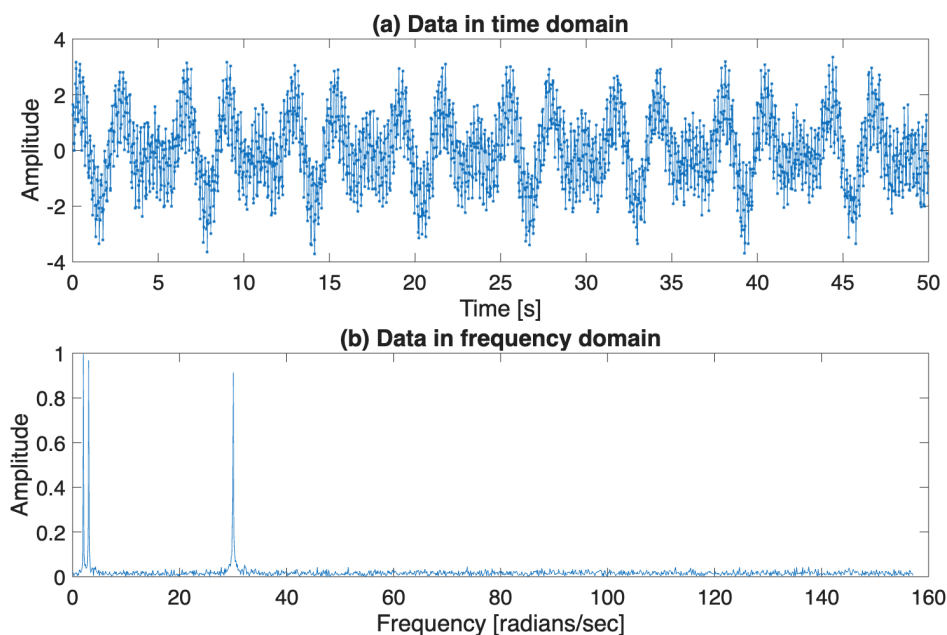


Figure 10: This is a partial reproduction of a figure from HW 13, provided here for informational purposes only.

Your task is to determine what the corresponding frequency-domain plot would look like if `fft` were used on the function shown in Figure 11. Make a sketch on Figure 12.

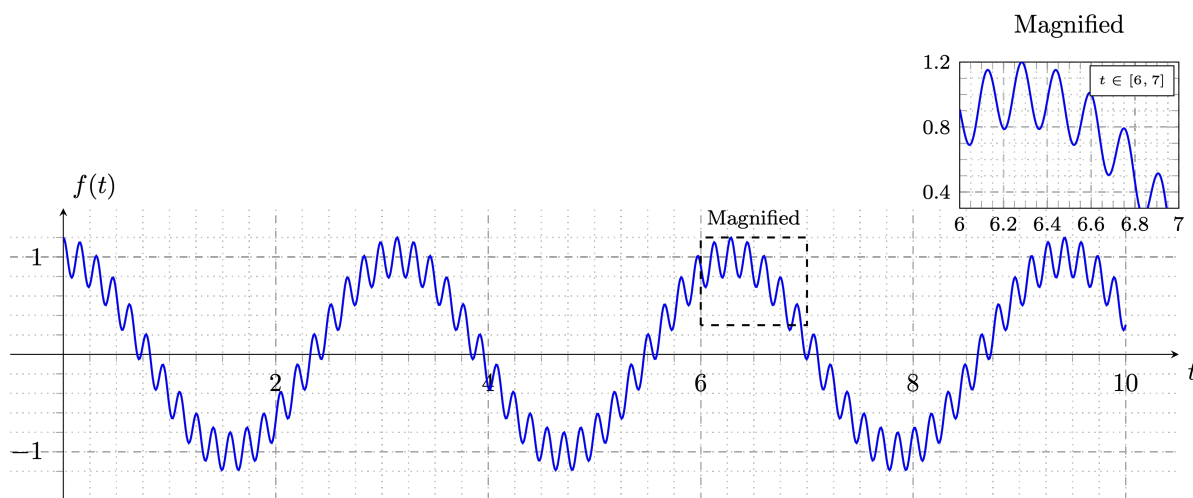


Figure 11: A time-varying signal containing multiple frequencies

i Note

The numerical *values* on the vertical axis of Figure 12 are not important, but the relative values of different peaks **is** important. The numerical value on the horizontal axis **is** important.

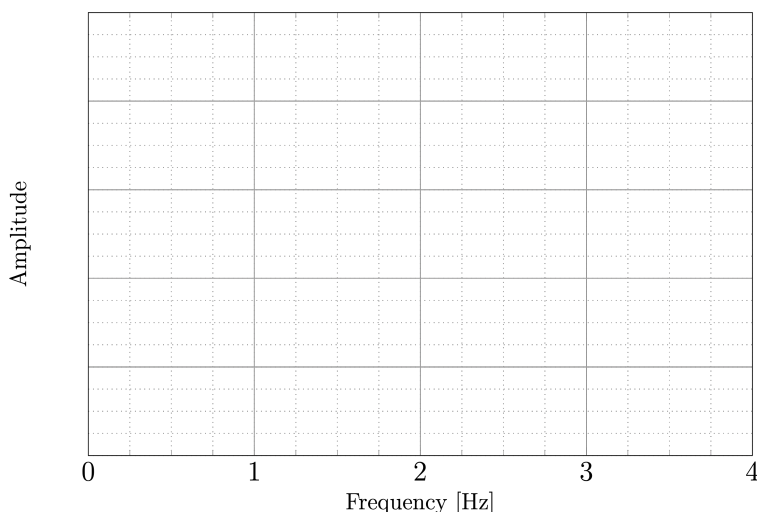


Figure 12

8.2 Bode Plot as filter

We would like to design a first-order linear system such that when the signal $f(t)$ from Figure 11 is provided to it as input, the high-frequency signal is attenuated (i.e., reduced in amplitude) and the low-frequency signal is maintained at or near the original amplitude. Draw the Bode plot for such a first-order linear system on the axes shown in Figure 13, identify the corner frequency, and write down the transfer function associated with this linear system.

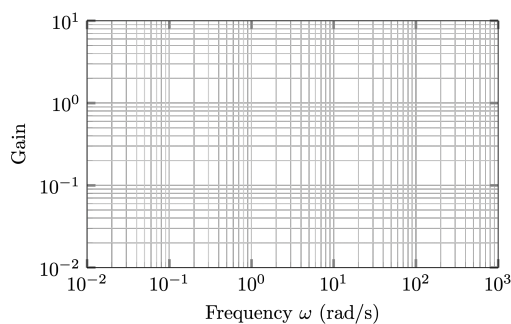


Figure 13

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