Problem Set 14

ENGR 12, Spring 2026.

Instructions

Date	Thu, Apr 30, 2026
Duration	One hour (60 minutes)

Name

Partner 1

Partner 2

- Write your answers in the spaces provided.
- Do not tear pages; Equation 1 is written on the header on every page for your convenience.
- You are **not allowed to use a calculator of any kind**, including handheld calculators, scientific calculators, Google Search, or your computer's calculator.
- You may access any part of the course website during this assignment.
- You have approximately one hour to work on this assignment.
- Write the names of your group members, if any, on the front page.

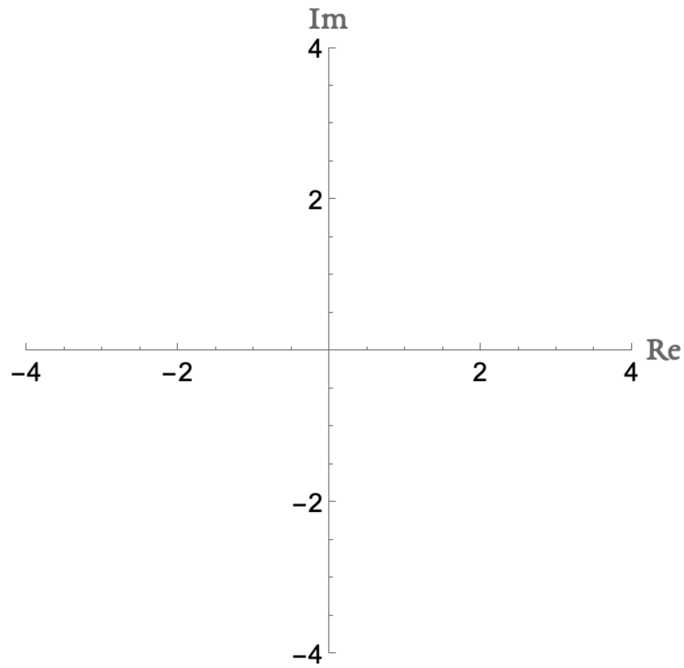
Tasks

In this assignment, you will analyze a system defined by the following differential equation

$$\boxed{2\ddot{x} + 7\dot{x} + 3x = f(t)} \tag{1}$$

1) Roots of the characteristic polynomial 2) Transfer Function 3) Step response 4) Impulse response 5) Bode plot 6) Amplitude and Phase Shift

1. Plot the roots of the characteristic polynomial of Equation 1 on the following set of axes. Then, determine if this system is stable or unstable and whether it is underdamped, overdamped, or critically damped.



2. Write down the transfer function of this system, where $F(s)$ is the input and $X(s)$ is the output. Do not use any other expression for the transfer function of second-order systems that you may be aware of; deduce the transfer function directly from Equation 1.

3. Find the unit step response of this system, and give your answer as a function of time.

4. Find the unit impulse response of this system, and give your answer as a function of time.

5. Make a quantitatively accurate sketch, by hand, of the Bode plot for this system, and draw vertical lines showing the corner frequencies.

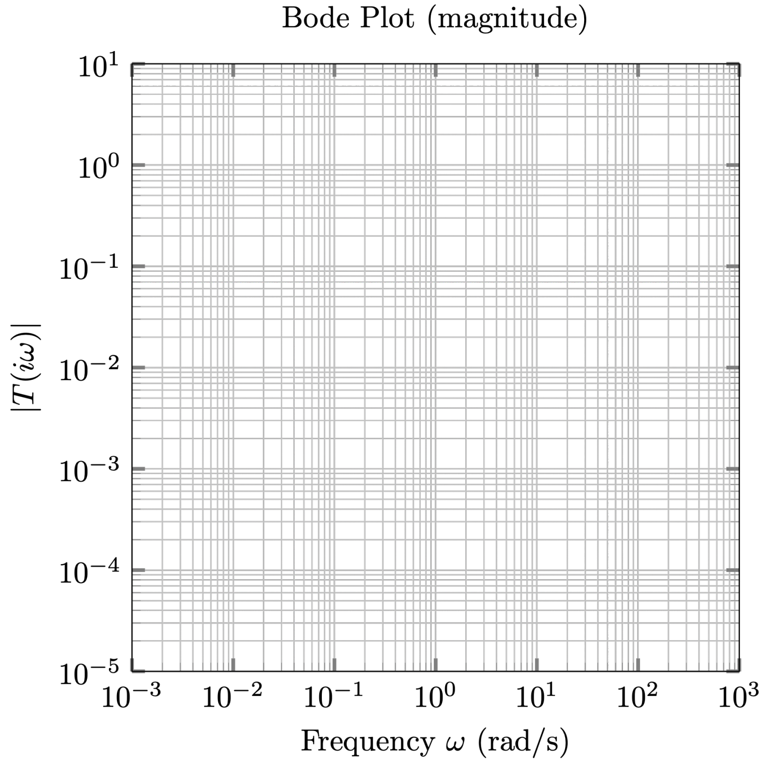


Figure 1: Note that the tick marks correspond to regularly-spaced intervals, i.e., the first tick mark after 10^{-1} is 0.2, the next is 0.3, and so on.

6. If the system is subjected to an input $f(t) = \sin t$, determine a mathematical expression for the output $x(t)$ after it has reached steady-state.