Problem Set 9

ENGR 12, Spring 2026.

Due Date	Fri, Apr 03
Turn in link	Gradescope
URL	`emadmasroor.github.io/E12-S26/Homework/HW9`

Points Distribution

Please note that each of the following grade items is a **single rubric item**. Each rubric item is scored on a four-level scale of 3-2-1-0. You may wish to take this into account when deciding how to allocate your efforts to each problem.

Problem	% Weightage
Problem 1	25
Problem 2	25
Problem 3	25
Problem 4	25

1 Forced Response of First-order systems

Consider a single-input, single-output linear system whose transfer function is given by

$$\frac{1}{4s+5}.$$

We would like to determine the response of this system to an input given by the following function of time.

$$\begin{cases} 2+3t & t>0 \\ 0 & t\leq 0 \end{cases}$$

1.1 Frequency domain

Use the formulae for the coefficients of partial fractions, given in lecture 18, to determine the response in the frequency domain as the sum of three terms.

1.2 Time domain

Use the table of Laplace Transforms to determine the response as a function of time.

1.3 Steady-State vs Transient

1. Write down the steady-state portion of the forced response, in the time domain and the frequency domain.
2. Write down the transient portion of the forced response, in the time domain and the frequency domain.

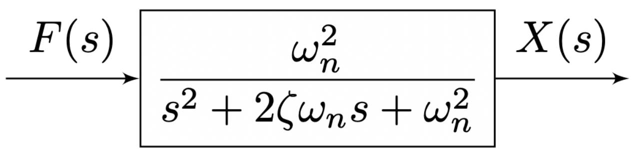
1.4 MATLAB

Use Matlab's `tf`, `step`, and `impulse` functions to generate plots of this system's step response and impulse response.

> How to submit
>
> Export the MATLAB figures as images and embed into your homework submission. You do not need to make any improvements to the figures, and you should need to type less than thre lines of code to generate the figures.

2 Second-order systems

Consider the system described by the block diagram shown below.



2.1 Underdamped case

If the system is **underdamped**, show that the poles of the transfer function are located at

$$-\zeta\omega_n \pm i\omega_n\sqrt{(1-\zeta^2)}$$

2.2 Overdamped case

If the system is **overdamped**, find the poles of the transfer function in terms of ω_n and ζ.

2.3 Impulse Response

For the following parts, use $\zeta = 2$ and $\omega_n = 3$.

2.3.1 Frequency domain

Find an expression in terms of s for the impulse response of this system in the frequency domain. Leave your answer in terms of square roots, i.e., do not simplify, e.g., $\sqrt{2}$ as 1.41.

> Hint #1
>
> Your answer should be the sum of two fractions having the form $\dfrac{a}{s-b}$.

> Hint #2
>
> 'the system's [blank] is also the system's impulse response'

2.3.2 Time domain

Use the table of Laplace Transforms to write down the impulse response of this system in the time domain, and use plotting software to generate a plot of this function against time.

3 Stability

Decide which of the following systems are stable, unstable, or marginally stable. Write a short explanation of how you arrived at your conclusion for each, **together with a plot** on the complex plane of the roots of the characteristic polynomial of each system.

1. A system described by the differential equation

$$2\ddot{x} + 3\dot{x} + 5x = 0$$

2. A system described by the differential equation

$$3\ddot{x} - 2\dot{x} + 2x = 0$$

3. A system described by the transfer function

$$\frac{1}{(2s+3)(3s-1)}$$

4. A system described by the transfer function

$$\frac{1}{(s+4)^2 + 5^2}$$

5. A system described by the transfer function

$$\frac{1}{s^2 + 7^2}$$

6. A system described by the transfer function

$$\frac{2s+3}{(s+2-i)(s+2+i)}$$

7. A system described by the differential equation $2\dot{x} + 5x = 2t$

4 Poles

4.1 Partial Fraction Expansions

Use the formulae derived in lecture 18 to determine the numerators of the following partial fraction expansions.

1.
$$\frac{1}{s^2(3s+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+4/3}$$

2.
$$\frac{1}{s^2+s+7} = \frac{A}{s-s_1} + \frac{B}{s-s_2}$$
where you determine what s_1 and s_2 are as well as the numerators A and B.

> **Hint**
>
> A and B may be complex numbers, and if so, you should think about the best way to represent these complex numbers.

4.2 Inverse Laplace Transform

Use the procedure from lecture 18 to determine the inverse Laplace transform of the following functions. Your answer should be a function of time.

1. For the following function
$$F(s) = \frac{2s-3}{\left(s+\frac{1}{2}\right)^2 + 27},$$
find $f(t)$ in terms of a function that uses both sin and cos.
2. For the following function
$$F(s) = \frac{1}{s^2+2s+3},$$
find $f(t)$ in terms of a function that uses sin and does not use cos.