

# E12 Lab 5

Frequency Response, Fourier Series, and Superposition

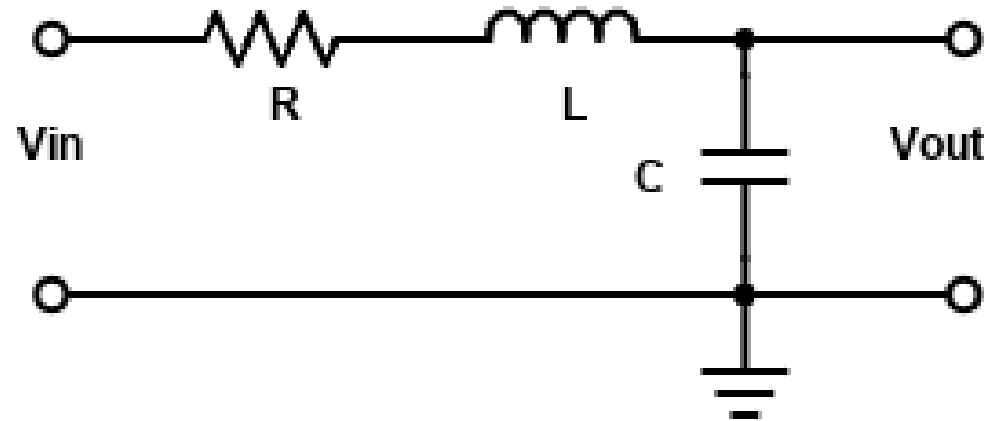
# Today we'll analyze this circuit

$$LC\ddot{v}(t) + RC\dot{v}(t) + v(t) = v_{in}(t)$$

$$T(s) = \frac{1}{LCs^2 + RCs + 1}$$

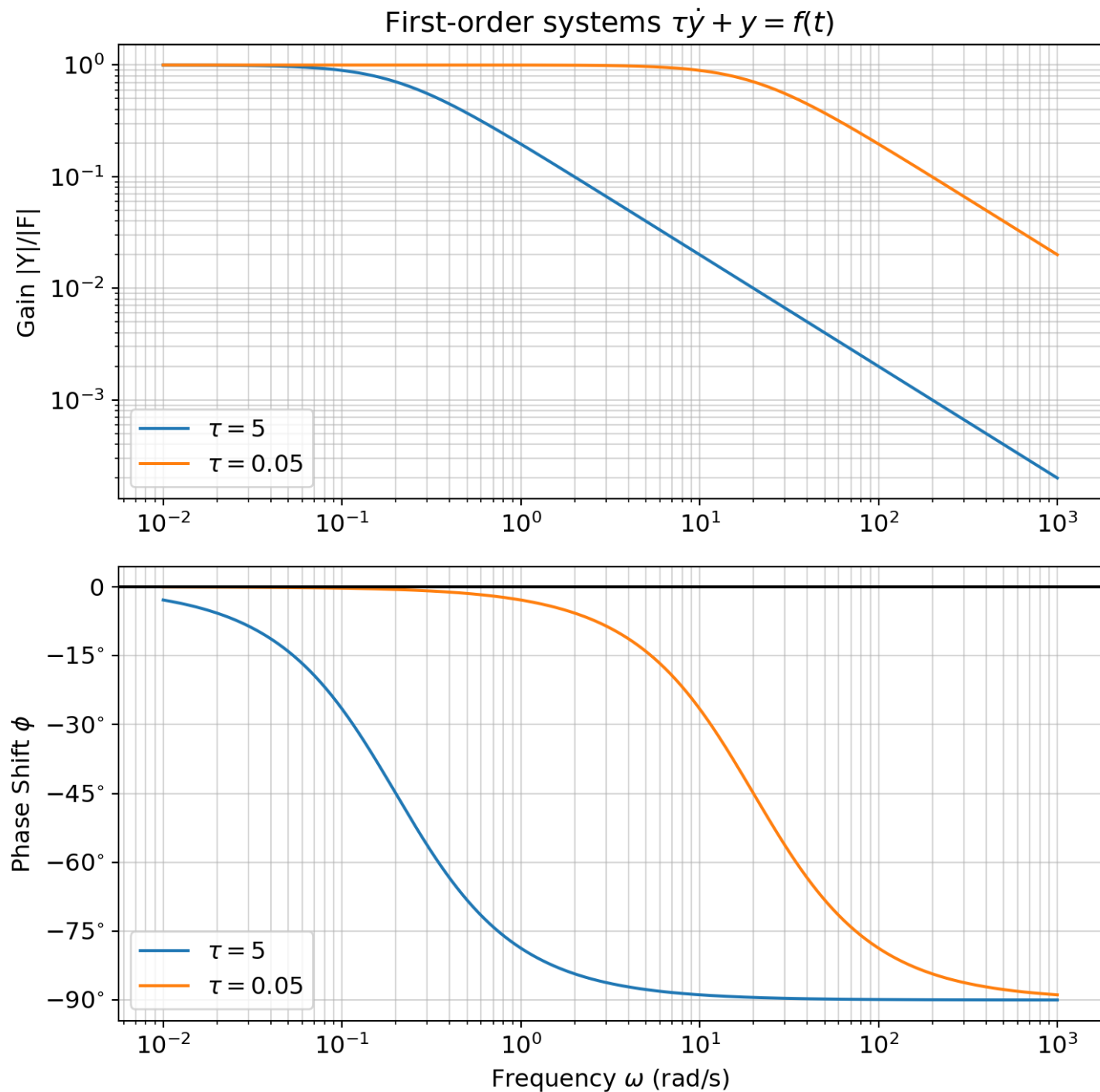
$$T(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0s + \omega_0^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ and } \zeta = \frac{R}{2} \sqrt{\frac{C}{L}}$$



# Recall bode plots

- Top plot shows the gain (attenuation)
- Bottom plot shows the phase offset
- Usually on a log scale



# Definition of a decibel (dB)

In electrical circuits, we're often interested in the power ratio (gain) of the output to the input or some reference power. It often makes sense to talk about this ratio on a logarithmic scale.

$$\text{bel} = \log_{10} \left( \frac{P_{out}}{P_{in}} \right)$$

A decibel is  $1/10^{\text{th}}$  of a bel. Often, we have a direct measurement of voltage instead of power. Knowing that  $P = V^2/Z$ , a decibel is...

$$\text{dB} = 10 \log_{10} \left( \frac{V_{out}^2/Z}{V_{in}^2/Z} \right) = 20 \log_{10} \left( \frac{V_{out}}{V_{in}} \right)$$

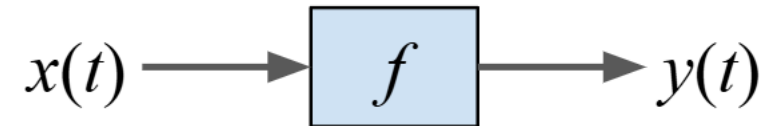
# Practice using decibels

$$\text{dB} = 10 \log_{10} \left( \frac{P_{out}}{P_{in}} \right) = 20 \log_{10} \left( \frac{V_{out}}{V_{in}} \right)$$

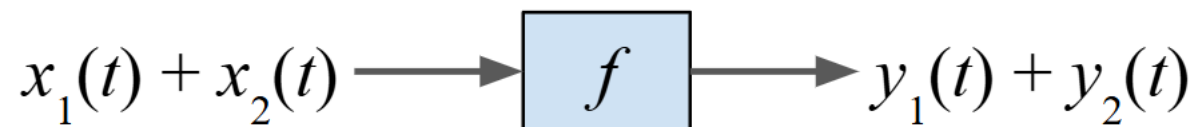
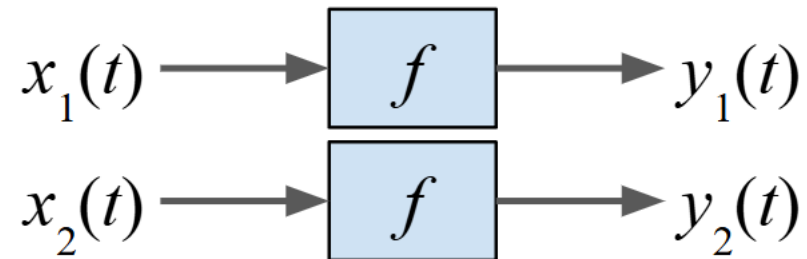
1. What is the gain in dB for a signal whose power is reduced by half?
2. What is the voltage ratio for a signal with an attenuation of  $-3$  dB?
3. What is the gain in dB for a signal whose voltage is reduced by half?
4. What is the attenuation in terms of voltage and power for a gain of  $0$  dB?

# Superposition in linear systems

Linear systems transform a time-varying input into a time-varying output



The response to the sum of inputs is the sum of the responses to individual inputs



# Fourier series

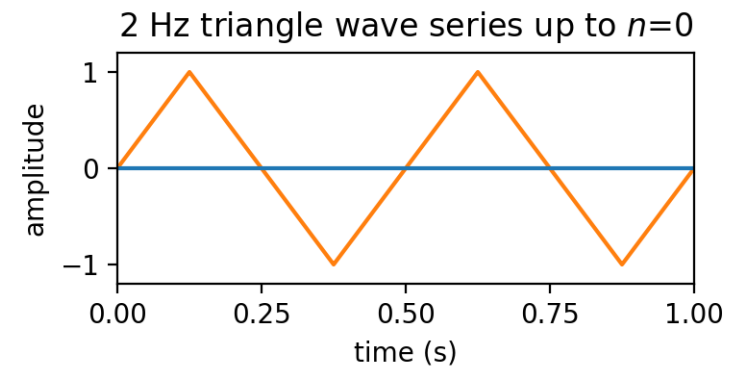
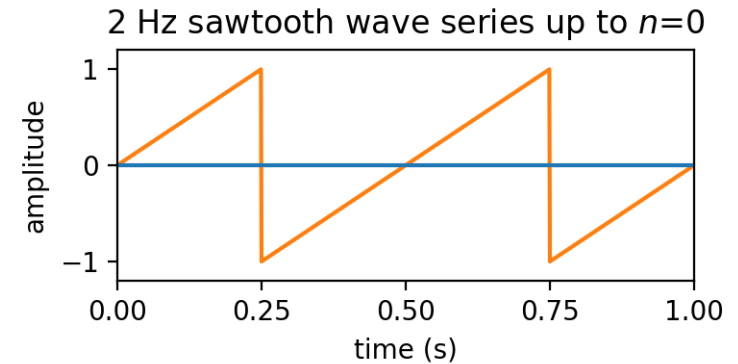
Approximates a periodic function  $x(t)$  with period  $P$  as a sum of sines and cosines.

$$x(t) \approx a_0 + \sum_{n=1}^N a_n \cos\left(\frac{2\pi n t}{P}\right) + b_n \sin\left(\frac{2\pi n t}{P}\right)$$

where  $a_0 = \frac{1}{P} \int_P x(t) dt$

$$a_n = \frac{2}{P} \int_P x(t) \cos\left(\frac{2\pi n t}{P}\right) dt \quad \text{for } n \geq 1$$

$$b_n = \frac{2}{P} \int_P x(t) \sin\left(\frac{2\pi n t}{P}\right) dt \quad \text{for } n \geq 1$$

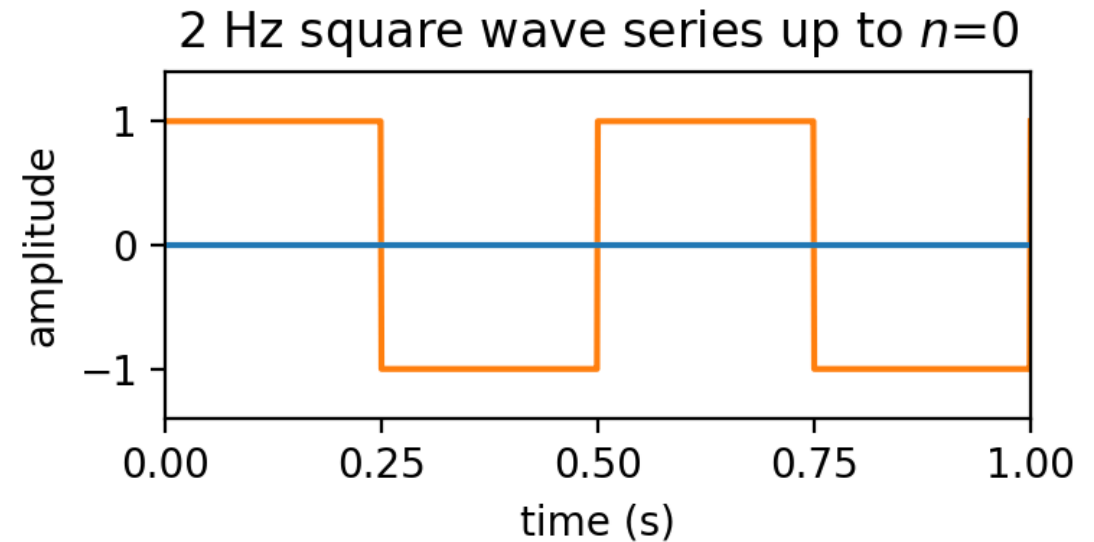


# Fourier series of a square wave

$$a_0 = 0$$

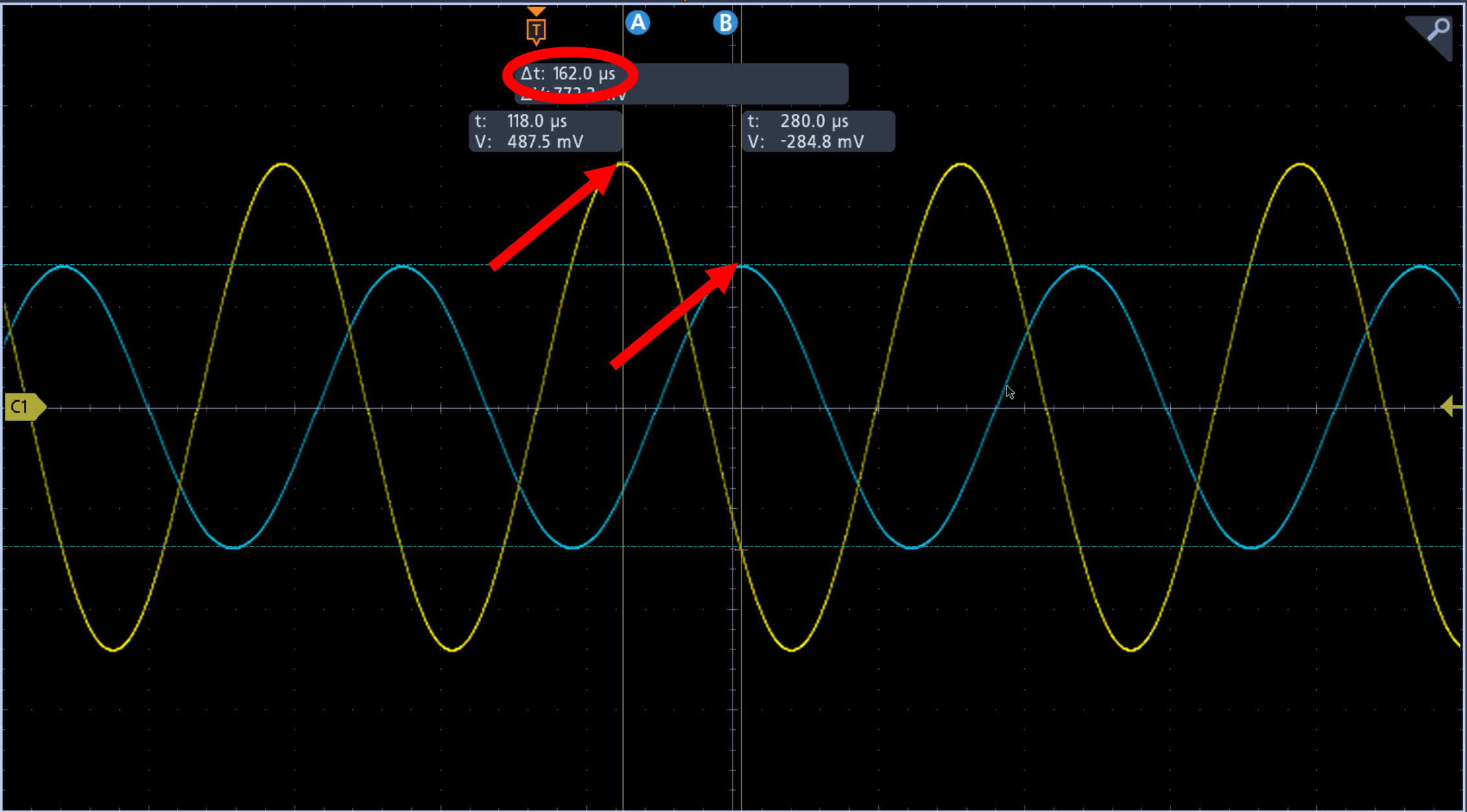
$$a_n = 0 \quad \text{for all } n \geq 1$$

$$b_n = \begin{cases} \frac{4}{n\pi} & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}$$



# Putting it all together

- We may not know how a circuit responds to an arbitrary input waveform. But, if we know:
  - how it responds to a sinusoidal input of different frequencies
  - the Fourier series that approximates the input waveform
- ...then the superposition principle gives us an excellent approximation for the output waveform!
- Today, you will prove that a square wave is the sum of sine waves of different frequencies by using the RLC circuit as a filter
  - Remove the high-frequency harmonics
  - Pass the fundamental frequency of the square wave



$\Delta t: 162.0 \mu s$   
 $V: 772.2 mV$   
 t: 118.0  $\mu s$   
 V: 487.5 mV  
 t: 280.0  $\mu s$   
 V: -284.8 mV

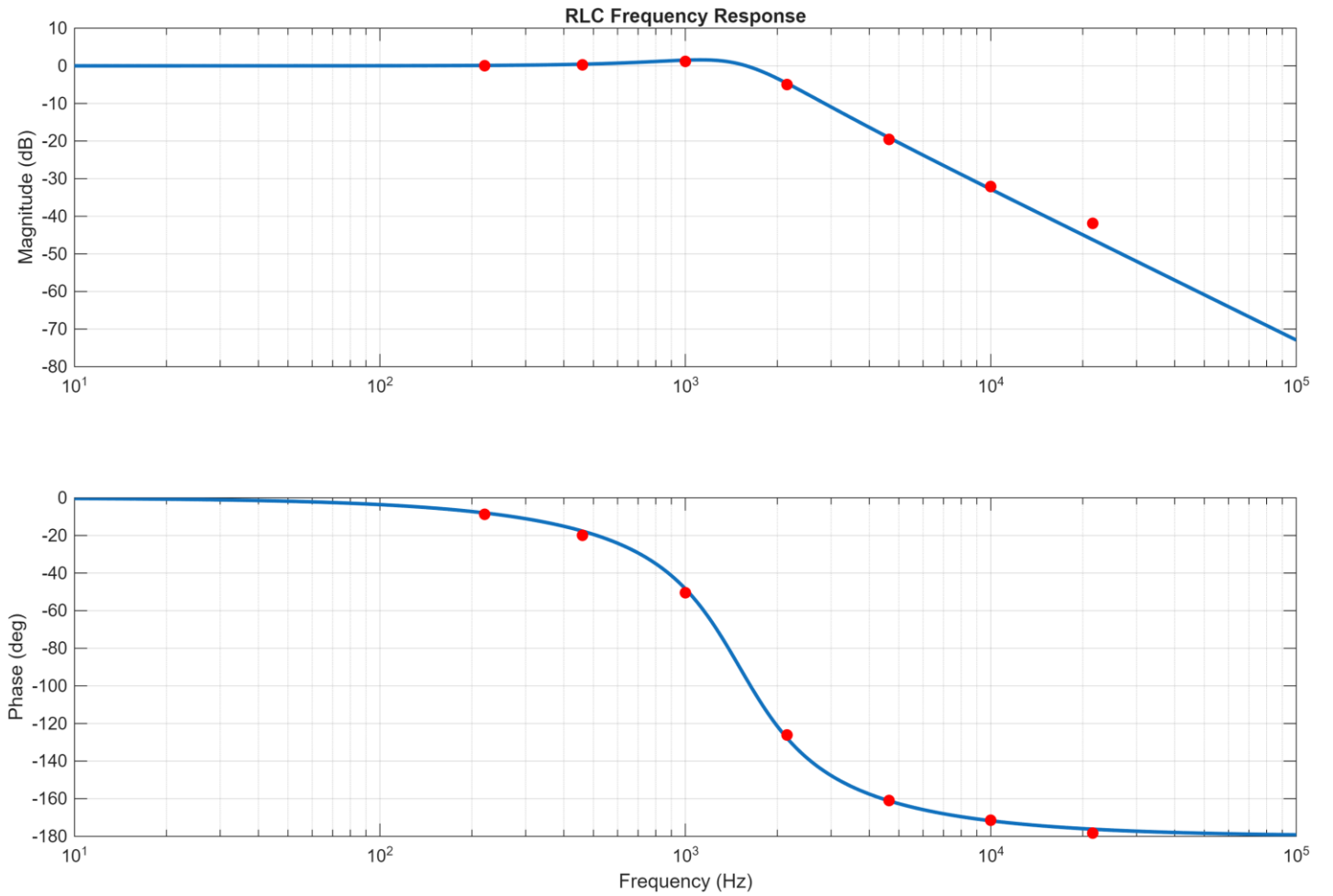
Cursors	Measure
Search	Results Table
Meas 1	1
High	488.7 mV
Meas 2	1
Low	-480.5 mV
Meas 3	1
Peak-Peak	969.3 mV
Meas 4	2
Peak-Peak	562.1 mV

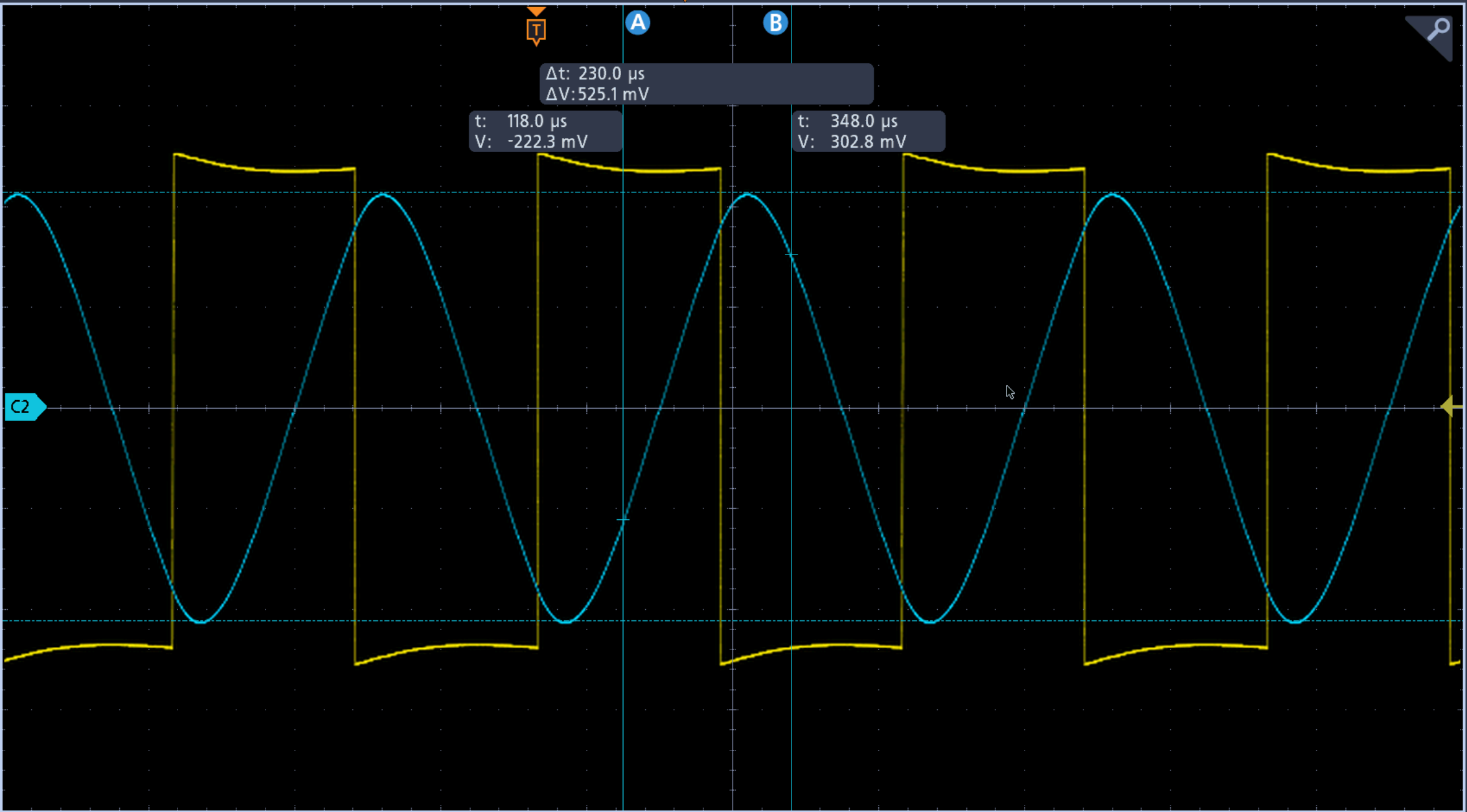
**Ch 1**  
 200 mV/div  
 1 M $\Omega$   
 100 MHz

**Ch 2**  
 200 mV/div  
 1 M $\Omega$   
 100 MHz

3 4 Math Ref Bus DVM AFG  
 Horizontal 200  $\mu s$ /div SR: 5.00 MS/s RL: 10 kpts  
 Trigger 1 4.00 mV  
 Acquisition High Res 1.162 MAcqs  
 Triggered  
 14 Apr 2026 17:04:57

Add  
experimental  
data to the  
theoretical  
bode plot





Cursors Measure

Search Results Table

Meas 1 1  
High  
472.0 mV

Meas 2 1  
Low  
-472.0 mV

Meas 3 1  
Peak-Peak  
1.016 V

Meas 4 2  
Peak-Peak  
852.9 mV

Ch 1  
200 mV/div  
1 MΩ  
100 MHz

Ch 2  
200 mV/div  
1 MΩ  
100 MHz

3 4 Math Ref Bus DVM AFG

Horizontal  
200 μs/div  
SR: 5.00 MS/s  
RL: 10 kpts

Trigger 1  
4.00 mV

Acquisition  
High Res  
1.135 MAcqs

Triggered

14 Apr 2026  
17:03:50