ENGR 21
Computer Engineering Fundamentals

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Lecture 17
Tuesday, Nov 4, 2025

Optimization

Optimization

Find the 'most ideal' design, configuration, inputs, etc.

$$f(X_1, X_2, X_3, \ldots, X_n) = \underbrace{\text{objective function}}$$

cast it as a minimization or maximization problem.

Maximize the fuel efficiency of an automobile

Independent variables:

- Materials
- Fuel type
- "Aerodynamics" of design
- Exhaust system

Objective function:

- Miles per gallon

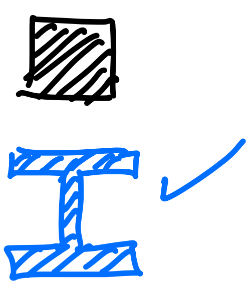
Constraints:

- Cost

Maximize the bending rigidity of a beam for unidirectional loading

Independent variables:

- Materials
- Spatial distribution of beam cross-section → "shape"



Objective function:

- Bending rigidity EI

Constraints:

- Total cross-sectional area

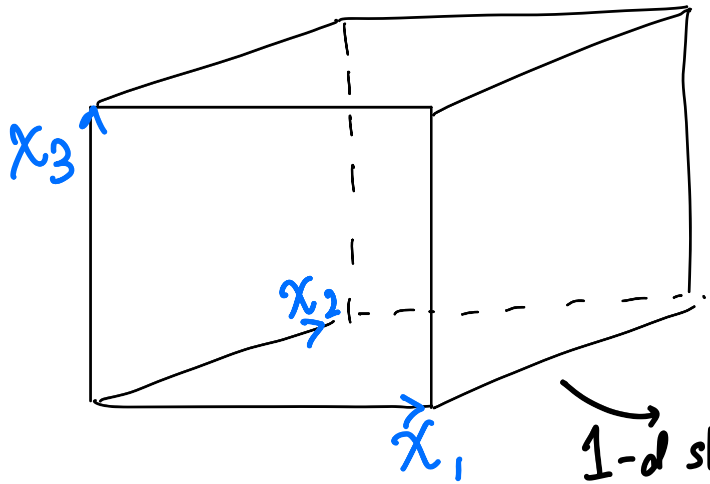
Finding a minimum (or maximum) in design-variable space

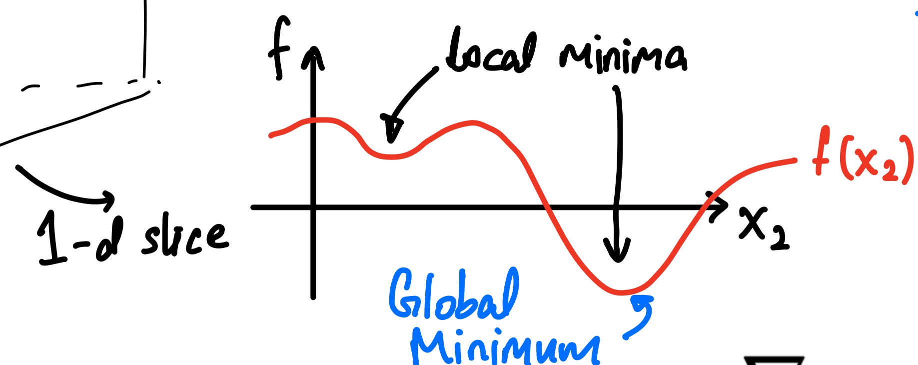
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \nabla = \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{bmatrix}$$

x_i : i^{th} configuration variable

$f(x_1, x_2, \ldots, x_n)$

$$\overbrace{\nabla}^{\text{gradient}} \underbrace{f}_{\text{scalar function}} = 0$$





$$\nabla \equiv \begin{bmatrix} \partial/\partial X_1 \\ \partial/\partial X_2 \\ \partial/\partial X_3 \\ \vdots \\ \partial/\partial X_n \end{bmatrix}$$

Global minimum

The set $\{x_1 = ?, x_2 = ?, \ldots x_n = ?\}$ that gives the smallest value of f anywhere.

Local minimum

The set $\{x_1 = ?, x_2 = ? \ldots x_n = ?\}$ that gives the smallest value in the neighborhood.

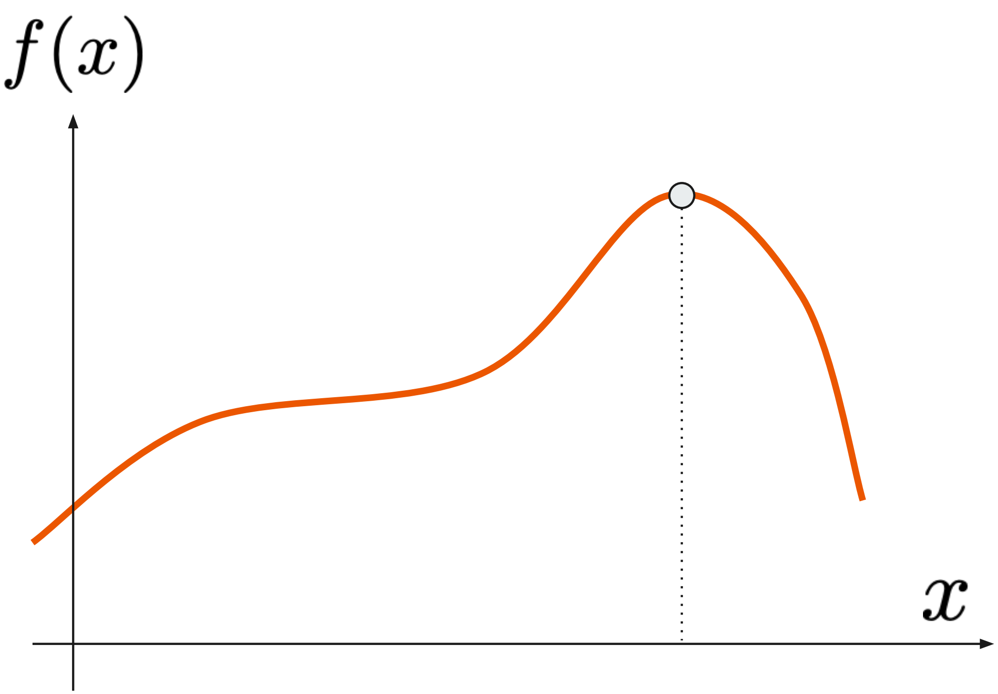
Techniques for single-variable unconstrained optimization

in practice, f could be so 'expensive' to compute that you can't easily solve the math problem $f'(x)=0$

Look for x, s.t. $f(x)$ = max.

$\Updownarrow$

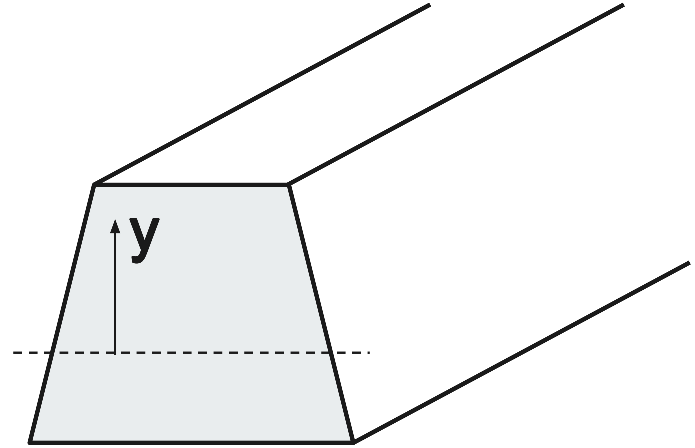
Look for x, s.t. $f'(x) = 0$



Methods that don't rely on knowledge of $f'(x)$ to maximize $f(x)$

An optimization problem

The shear stress on the face of this beam, in terms of distance from its centroid y, is hard to calculate.



Shear Stress =

$$\frac{61224.2 - 1.9334\times10^{6}\,y - 7.01705\times10^{8}\,y^{2} + 4.375\times10^{10}\,y^{3} - 6.81818\times10^{11}\,y^{4}}{0.000686111 - 0.0533333\,y + 1.\,y^{2}}$$

If you are only allowed to calculate the above formula a total of 10 times, how would you determine the value of y where the shear stress is a maximum?

Lec 9.1 Thu, Nov 4

Optimize the following function

```
def shearStress(y):
    return (61224.2 - 1.9334e6*y - 7.01705e8 * y**2 + 4.375e10 * y**3 - 6
```

(Looking for a maximum in $f(x)$), x^*

Bracketing for single-variable unconstrained optimization of f(x)

first step

and check $f(x)$ each time

1. Pick some x
2. Increment $x_0 \rightarrow x_1 \rightarrow x_2 \rightarrow x_3 \rightarrow$ ... in direction of increasing f(x)
3. Keep going until f(x) decreases → keep the last 2 points

$$[x_{n-1}, x_n]$$

maximum, x^*, has been successfully bracketed inside this interval

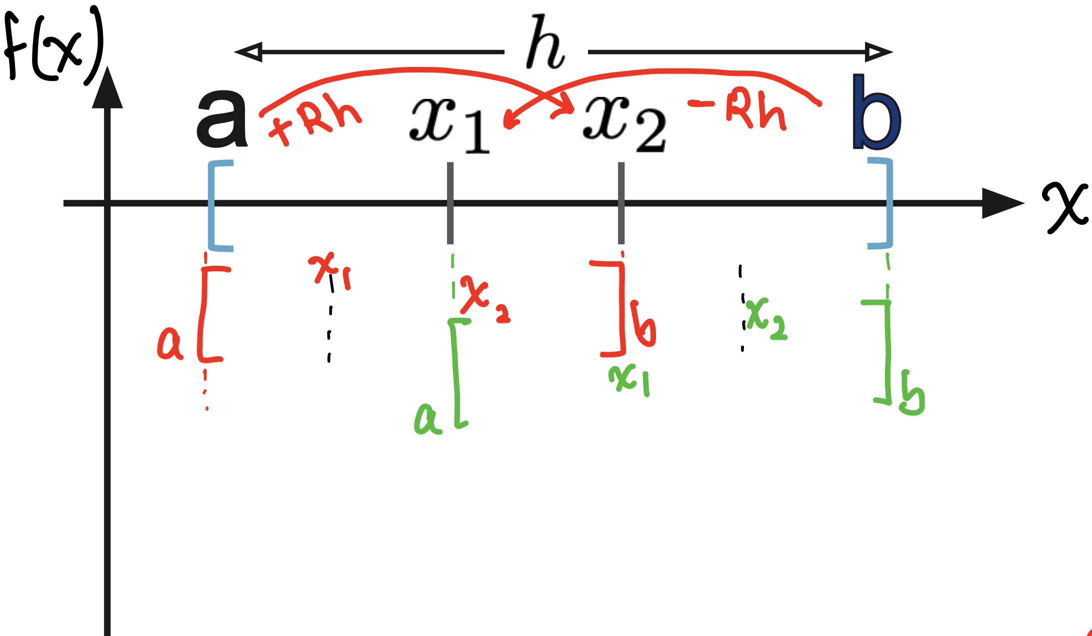
Incrementing strategies:

- constant increment $x_{i+1} = x_i + 0.1$, e.g.
- increasing increment $x_0 = 0.1$
  $x_1 = 0.2$
  $x_2 = 0.4$
  $x_3 = 0.8$ ...

After successful bracketing, need more sophisticated methods:—

Golden Section Search for single-variable unconstrained optimization of f(x)

after we know a bracket [a,b] between which the optimum lies.



Choose two special points inside the bracket:

$$x_1 = b - Rh$$

$$x_2 = a + Rh$$

$$R = \frac{-1+\sqrt{5}}{2} \approx 0.618...$$

Calculate f(x) at x_1, x_2 (already know f(a), f(b))

Check if $f(x_1) > f(x_2) \rightarrow$ new bracket $[a, x_2]$

if $f(x_1) < f(x_2) \rightarrow$ new bracket $[x_1, b]$

Next: repeat

Goal: successively narrow down the interval where f is minimized

$R = \frac{-1+\sqrt{5}}{2}$

if $f(x_1) > f(x_2) \rightarrow$ new bracket $[a, x_2]$

if $f(x_1) < f(x_2) \rightarrow$ new bracket $[x_1, b]$

$x_1 = b - Rh$

$x_2 = a + Rh$

$a \quad x_1 \quad x_2 \quad b$

$[\quad \vdots \quad \vdots \quad]$

Maximize f(x) = sin(2x) in the region [0,1]

is it $[x_1, b]$ or $[a, x_2]$

Step	a	x_1	x_2	b	h	new interval	
0	0.0	$x_1 = 0.3819$ $f(x_1) = 0.691$ repeat	$x_2 = 0.618$ $f(x_2) = 0.944$	1.0	1	$[x_1, b]$	
1	0.3819	$x_1 = 0.618$ $f(x_1) = 0.944$	$x_2 = 0.763$ $f(x_2) = 0.998$	1.0	0.618		