Worksheet for Gaussian Elimination

In this problem, you will perform 'Gaussian elimination' by hand. Given the equation $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 2 & 1 & -1 & 0 \\ 1 & 1 & 2 & 0 \\ -1 & 2 & 1 & 1 \\ 6 & 1 & 1 & -2 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix},$$

carry out each step of the Gaussian elimination procedure by writing the intermediate 'augmented' matrices. There are a total of six steps in this process; each step 'gets rid of' one term from the matrix A while introducing other changes. Each step takes the form

$$\text{Let row } ____ = \text{row } ____ - ____ \times \text{ row } ____$$

and is followed by an updated augmented matrix of the form

$$\left[\begin{array}{cccc|c} A_{11} & A_{12} & A_{13} & A_{14} & b_1 \\ A_{21} & A_{22} & A_{23} & A_{24} & b_2 \\ A_{31} & A_{32} & A_{33} & A_{34} & b_3 \\ A_{41} & A_{42} & A_{43} & A_{44} & b_4 \end{array}\right],$$

where some entries will be zero. Do this problem in the template given below.

1. Let row 2 = row 2 $- (a_{21}/a_{11}) \times$ row 1.
   The new augmented matrix is now

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 1 \\ 0 & 0.5 & 2.5 & 0 & -1.5 \\ -1 & 2 & 1 & 1 & 0 \\ 6 & 1 & 1 & -2 & 2 \end{array}\right]$$

2. Let row 3 = row 3 $- (a_{31}/a_{11}) \times$ row 1.
   The new augmented matrix is now

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 1 \\ 0 & 0.5 & 2.5 & 0 & -1.5 \\ 0 & 2.5 & 0.5 & 1 & 0.5 \\ 6 & 1 & 1 & -2 & 2 \end{array}\right]$$

3. Let row 4 = row 4 $- (a_{41}/a_{11}) \times$ row 1.
   The new augmented matrix is now

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 1 \\ 0 & 0.5 & 2.5 & 0 & -1.5 \\ 0 & 2.5 & 0.5 & 1 & 0.5 \\ 0 & -2 & 4 & -2 & -1 \end{array}\right]$$

4. Let row 3 = row 3 $-$ $(a_{32}/a_{22})\times$ row 2.
   The new augmented matrix is now

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 1 \\ 0 & 0.5 & 2.5 & 0 & -1.5 \\ 0 & 0 & -12 & 1 & 8 \\ 0 & -2 & 4 & -2 & -1 \end{array}\right]$$

5. Let row 4 = row 4 $-$ $(a_{42}/a_{22})\times$ row 2.
   The new augmented matrix is now

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 1 \\ 0 & 0.5 & 2.5 & 0 & -1.5 \\ 0 & 0 & -12 & 1 & 8 \\ 0 & 0 & 14 & -2 & -7 \end{array}\right]$$

6. Let row 4 = row 4 $-$ $(a_{43}/a_{33})\times$ row 3.
   The new augmented matrix is now

$$\left[\begin{array}{cccc|c} 2 & 1 & -1 & 0 & 1 \\ 0 & 0.5 & 2.5 & 0 & -1.5 \\ 0 & 0 & -12 & 1 & 8 \\ 0 & 0 & 0 & -5/6 & 7/3 \end{array}\right]$$