Worksheet for Gaussian Elimination

In this problem, you will perform 'Gaussian elimination' by hand. Given the equation $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 2 & 1 & -1 & 0 \\ 1 & 1 & 2 & 0 \\ -1 & 2 & 1 & 1 \\ 6 & 1 & 1 & -2 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 2 \end{bmatrix},$$

carry out each step of the Gaussian elimination procedure by writing the intermediate 'augmented' matrices. There are a total of six steps in this process; each step 'gets rid of' one term from the matrix A while introducing other changes. Each step takes the form

$$\text{Let row } _____ = \text{row } _____ - _____ \times \text{ row } _____$$

and is followed by an updated augmented matrix of the form

$$\left[\begin{array}{cccc|c} A_{11} & A_{12} & A_{13} & A_{14} & b_1 \\ A_{21} & A_{22} & A_{23} & A_{24} & b_2 \\ A_{31} & A_{32} & A_{33} & A_{34} & b_3 \\ A_{41} & A_{42} & A_{43} & A_{44} & b_4 \end{array}\right],$$

where some entries will be zero. Do this problem in the template given below.

1. Let row _____ = row _____ − _____ × row _____.

   The new augmented matrix is now

   $$\left[\begin{array}{cccc|c} & & & & \\ & & & & \\ & & & & \\ & & & & \end{array}\right]$$

2. Let row _____ = row _____ − _____ × row _____

   The new augmented matrix is now

   $$\left[\begin{array}{cccc|c} & & & & \\ & & & & \\ & & & & \\ & & & & \end{array}\right]$$

3. Let row _____ = row _____ − _____ × row _____

   The new augmented matrix is now



4. Let row _____ = row _____ − _____ × row _____

   The new augmented matrix is now



5. Let row _____ = row _____ − _____ × row _____

   The new augmented matrix is now



6. Let row _____ = row _____ − _____ × row _____

   The new augmented matrix is now

