Consider the following sets of simultaneous equations.

Set 1

Solve

$$4x_1 + 2x_2 = -3 \tag{1}$$
$$2x_1 + x_2 = 2 \tag{2}$$

for x_1 and x_2.

Set 2

Solve

$$4x_1 + 2x_2 = -3 \tag{3}$$
$$2x_1 - x_2 = 2 \tag{4}$$

for x_1 and x_2.

Classifying Matrices

In the following, identify:

1. the matrices that are likely to have a unique solution to $Ax = b$
2. the matrices that represent systems of equations that can be solved using the Jacobi method
3. the matrices that represent systems of equations that can be solved using LU factorization (after rearranging the rows if necessary)

$$B_1 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 3 & 2 \\ 0 & -2 & 1 \end{bmatrix}$$

$$B_3 = \begin{bmatrix} 2 & 1 & -1 & 3 \\ 0 & 3 & 1 & 4 \\ 4 & 2 & -2 & 6 \\ 1 & 0 & 2 & -1 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 2 & -1 & 4 & 1 \\ 1 & 1 & 2 & 1 \\ 2 & 0 & 2 & 3 \end{bmatrix}, \quad B_5 = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 3 & 1 & 0 \end{bmatrix}$$

For each of the following systems of equations,

1. Solve the equations for x and y.
2. Find the determinant of the associated matrix
3. Calculate the condition number.

> **Tip**
>
> Note that the matrix
>
> $$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
>
> has determinant $|ad - bc|$ and its inverse is
>
> $$\frac{1}{|ad - bc|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Set 1

$$2x + y = 3 \tag{5}$$
$$2x + 1.001y = 0 \tag{6}$$

Set 2

$$2x + y = 3 \tag{7}$$
$$2x + 1.002y = 0 \tag{8}$$

Set 3

$$2x + y = 3 \tag{9}$$
$$2x + 1.001y = -0.001 \tag{10}$$