Lab 5: Compression and Buckling

ENGR 059 ▪ Fall 2026 ▪ Swarthmore Engineering

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1 Theory

> **Note**
>
> The theory of beam buckling is covered in *Hibbeler, Mechanics of Materials* in Chapter 13, and in *Beer & Johnston, Mechanics of Materials* in Chapter 10.1

Materials that show elastic behavior in tension also tend to show elastic behavior in compression. In compression, an additional possible mode of failure — which is impossible in tension loading — is *buckling*.

Leonhard Euler derived a formula for the critical compressive load P_{cr} at which a column will buckle. This crticial load is

$$P_{\text{cr}} = \frac{\pi^2 EI}{L_e^2}, \tag{1}$$

where E is the Young's Modulus, I is the smaller of the two moments of inertia I_x and I_y of the cross section about its respective neutral axes, and L_e is the **effective length** of the column. The effective length is related to the true length by a multiplicative factor, i.e.,

$$L_e = KL \tag{2}$$

where K depends on the way the load is applied.

1. If both ends of the column are pin joints, $K = 1$.
2. If one end of the column is fixed and the other is free, $K = 2$.
3. If one end is fixed and the other is pinned, $K = 0.7$
4. If both ends are fixed, $K = 0.5$.

Using the relation between stress, force and area for uniaxial loading, it is possible to derive a formula for the critical normal stress at which buckling occurs, i.e.,

$$\sigma_{\text{cr}} \equiv \left(\frac{P_{\text{cr}}}{A}\right) = \frac{\pi^2 EI}{L_e^2} \div A = \frac{\pi^2 E \overbrace{(Ar^2)}^{I}}{L_e^2 \cdot A} = \frac{\pi^2 E}{(L_e/r)^2}, \tag{3}$$

where we have used the fact that

$$r \equiv \sqrt{\frac{I}{A}}$$

is the **radius of gyration** of the cross-section.

> **Warning**
>
> Note that r here is **not** the actual radius; it is a radius of gyration.

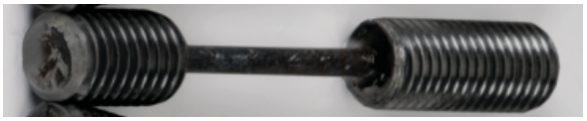
The ratio L_e/r is known as the **slenderness ratio**.

> **Condition for buckling**
>
> Buckling can only occur if
>
> $$\sigma_{\text{cr}} \leq \sigma_Y, \tag{4}$$
>
> where σ_Y is the yield stress and σ_{cr}, given by Equation 3, is the stress corresponding to the Euler load. This is a necessary, but sufficient, condition.

2 Materials and Methods

2.1 Samples

Table 1

Sample	Material	Image	PASCO Number
1 (x3)	Polyethylene		ME-8248
2 (x3)	ABS		ME-7012
3 (x3)	ABS		ME-7012
4 (x3)	ABS		ME-6987
5 (x3)	ABS		ME-6987
6 (x3)	ABS		ME-6987
7 (x3)	Annealed Steel		ME-8233

2.2 Material Properties

For metals, compressive material properties are not very different from tensile ones. However, for other materials such as plastics, the numbers can be significantly different.

The values below represent **estimates** only.

Material	Compressive Yield Strength	Compressive Young's Modulus
ABS	65 MPa	2.5 GPa
Polyethylene	20 — 80 MPa	0.7 — 1 GPa
Annealed Steel	345 MPa	200 GPa

2.3 Attachments

In this lab, you will use two attachments to the PASCO Materials Testing System.

Table 2

Type	Image	PASCO Number
Beam Fixture		ME-8242
Compression Accessory		ME-8247

3 Tasks

3.1 Calibration

Perform compliance calibration in compression.

3.2 Procedure

Load all three specimens of all samples in Table 1 in compression to failure, i.e., until:

- the material yields, or
- the beam buckles (let it buckle significantly, for fun!), or
- the load cell's limit of 7,000 N has been reached.

Record stress (P/A) and strain ($\Delta l/l_0$, where l_0 is the true length of the portion of the sample that is a uniform-cross section prism) using PASCO's 'Calculations' feature.

Take photographs of your **loaded** samples before, after and (if possible) during failure to document whether buckling occurs.

You may wish to consult Section 4 to ensure that you are collecting all necessary information for the memo.

4 Memo

Write a brief memo reporting on the results of your experiment. Your report should include:

1. High quality to-scale[1] drawings of the cross-section of each sample. Your drawings should be made in engineering software.
2. Stress-Strain diagrams for the three materials (ABS, annealed steel, and polyethylene). Combine curves where appropriate to present the information as succinctly as possible.
3. Photographs documenting buckling, when applicable.
4. For all samples in Table 1, report the following quantities using a table or tables.

    a. Length,
    b. Cross-sectional area,
    c. Second moment of area about the neutral axis, including both I_x and I_y, where the x and y axes represent the two principal directions in the cross-sectional plane,
    d. Radius of gyration in both x and y directions,
    e. The predicted critial load P_{cr},
    f. The predicted critical stress σ_{cr},
    g. The slenderness ratio L_e/r.
    h. Whether Equation 4 holds,
    i. Whether $P_{\text{cr}} < 7,000\,\text{N}$

    Optionally, you can also plot some of this information in charts, tables, or graphs.
5. A brief narrative, supported with numerical evidence from your tests, about how well buckling theory predicts actually-occuring buckling in various materials and geometries.

[1] Making a drawing *to scale* means that you should pick a certain scale (say, 1 inch on the paper represents 1/2 inch in reality; choose a scale that optimizes readability if your memo is printed.) and make *all* your drawings to the same scale. This allows accurate comparisons across drawings.