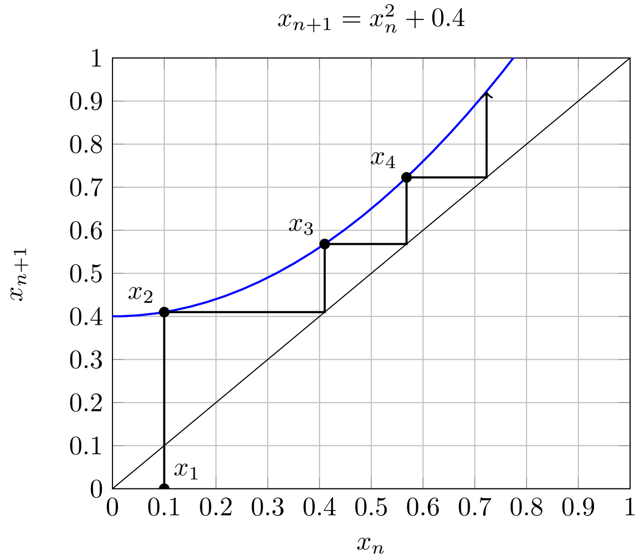
Consider the map given by

$$x_{n+1} = x_n^2 + 0.4, \tag{1}$$

starting from the initial condition $x_0 = 0.1$.

To *iterate* the map forward 4 steps — which you can think of as four discrete steps in time — it suffices to simply apply (1) to x_0 four times. The result is summarized in the table below, and visualized in the **cobweb diagram** to its left.



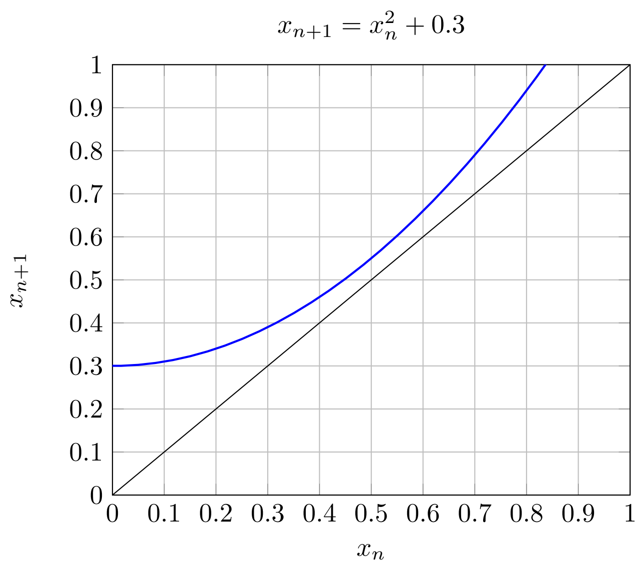
x_0		`0.1`
x_1	$= f(x_0)$	`0.41`
x_2	$= f(f(x_0))$	`0.5681`
x_3	$= f(f(f(x_0)))$	`0.722738`
x_4	$= f^4(x_0)$	`0.92235`

Now consider the map given by

$$x_{n+1} = f(x_n)$$
$$f(x) = x^2 + 0.3,$$

starting from $x_0 = 0.1$.

✍ Fill in the following table, and draw the cobweb diagram for this map.



x_0		`0.1`
x_1	$= f(x_0)$	
x_2	$= f(f(x_0))$	
x_3	$= f(f(f(x_0)))$	
x_4	$= f^4(x_0)$	
x_5	$= f^5(x_0)$	
x_6	$= f^6(x_0)$	
x_7	$= f^7(x_0)$	

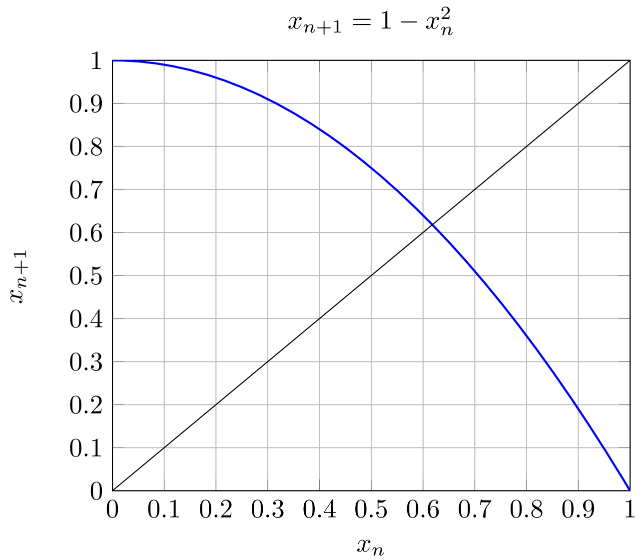
✍ Can you predict the long-term behaviour?

Consider the map given by

$$x_{n+1} = 1 - x_n^2, \quad (2)$$

starting from $x_0 = 0.5$.

✍ Fill in the following table, and draw the cobweb diagram for this map.

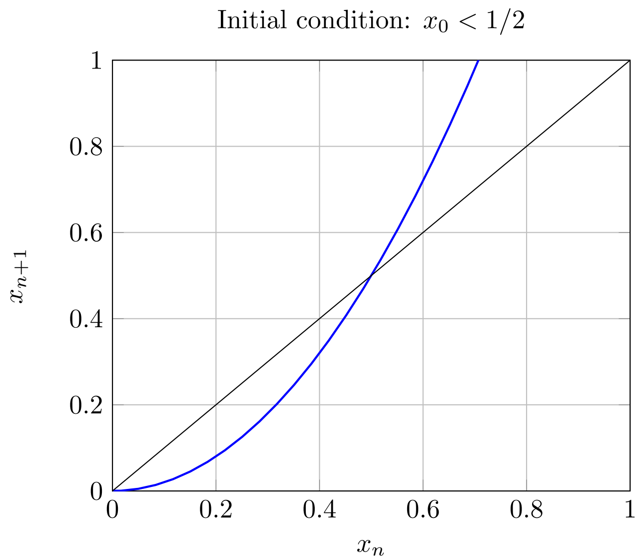


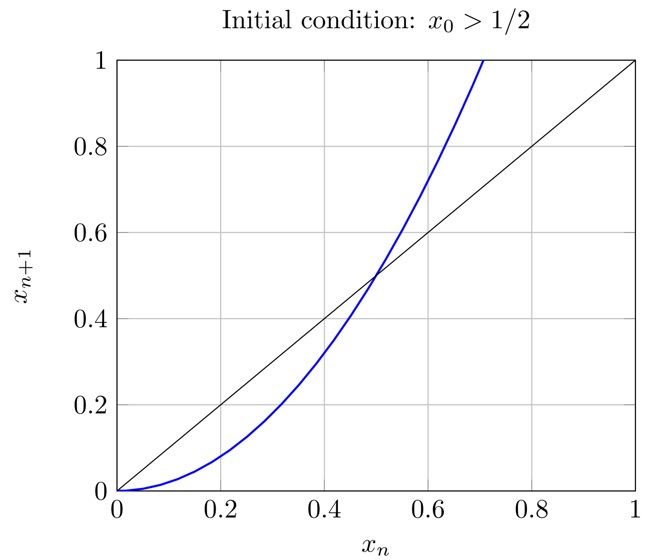
x_0		0.5
x_1	$= f(x_0)$	
x_2	$= f(f(x_0))$	
x_3	$= f(f(f(x_0)))$	
x_4	$= f^4(x_0)$	
x_5	$= f^5(x_0)$	

Next, consider the map given by

$$x_{n+1} = 2x_n^2. \quad (3)$$

✍ Draw a few steps of the cobweb diagram for this map using two different initial conditions.

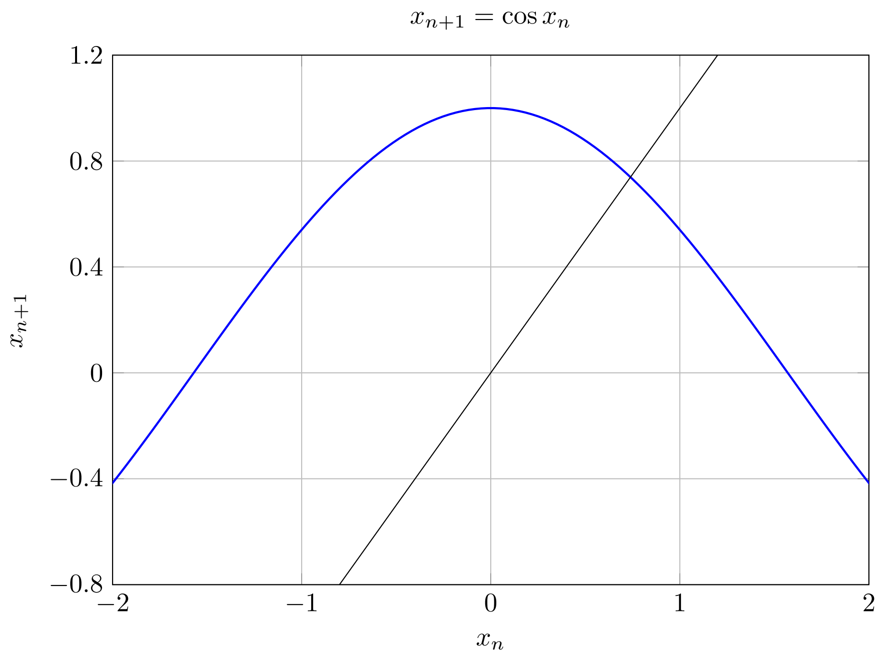




Now consider the map given by

$$x_{n+1} = cos x_n. \tag{4}$$

✍ Draw a few steps of the cobweb diagram for this map using $x_0 = -2$.

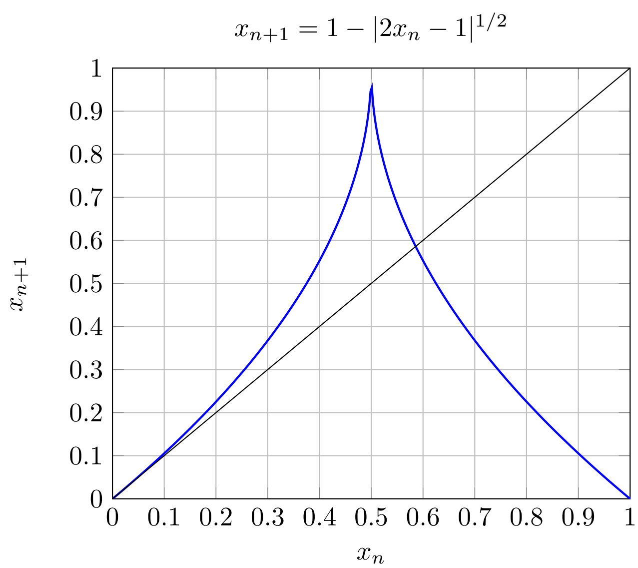


Now consider the map given by

$$x_{n+1} = 1 - |2x_n - 1|^{1/2} \tag{5}$$

starting from $x_0 = 0.25$.

✍ Fill in the following table, and draw the cobweb diagram for this map.



x_0		0.25
x_1	$= f(x_0)$	
x_2	$= f(f(x_0))$	
x_3	$= f(f(f(x_0)))$	
x_4	$= f^4(x_0)$	
x_5	$= f^5(x_0)$	
x_6	$= f^6(x_0)$	
x_7	$= f^7(x_0)$	

✍ Can you predict the long-term behaviour?