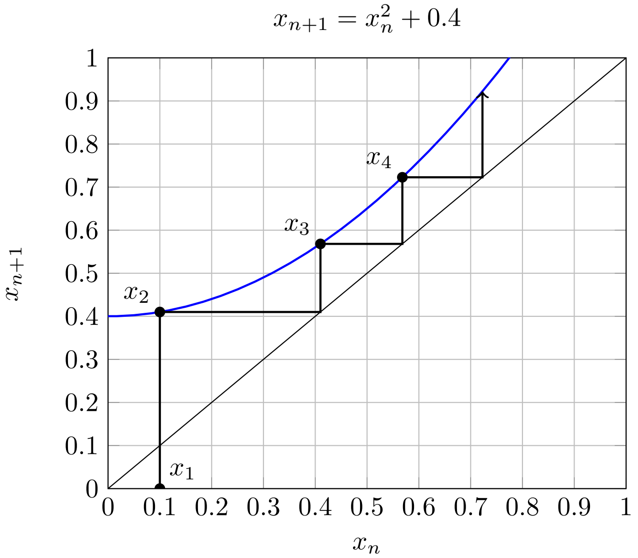
# Cobweb Diagrams and the Logistic Map

In this exercise, you will be making *cobweb diagrams*. Let's review how to do this. Consider the map given by

$$x_{n+1} = x_n^2 + 0.4, \tag{1}$$

starting from the initial condition $x_0 = 0.1$.

To *iterate* the map forward 4 steps — which you can think of as four discrete steps in time — it suffices to simply apply (1) to $x_0$ four times. The result is summarized in the table below, and visualized in the **cobweb diagram** to its left.



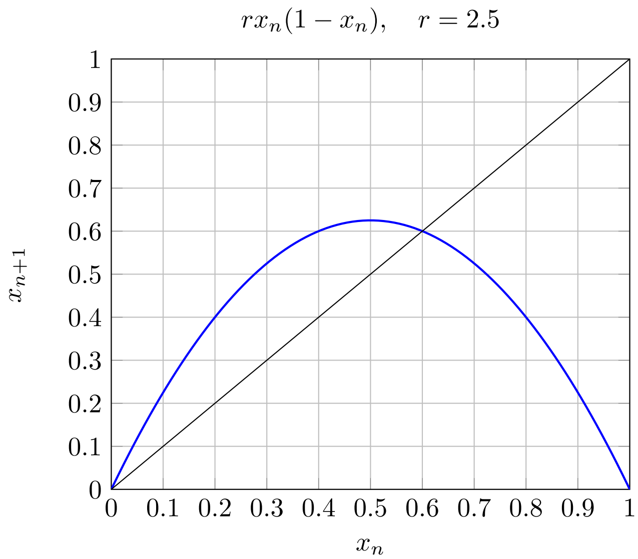

| | | |
|---|---|---|
| $x_0$ | | `0.1` |
| $x_1$ | $= f(x_0)$ | `0.41` |
| $x_2$ | $= f(f(x_0))$ | `0.5681` |
| $x_3$ | $= f(f(f(x_0)))$ | `0.722738` |
| $x_4$ | $= f^4(x_0)$ | `0.92235` |

Now consider the map given by

$$x_{n+1} = rx_n(1 - x_n)$$
$$r = 2.5,$$

starting from $x_0 = 0.3$.

✍ Fill in the following table, and draw the cobweb diagram for this map.

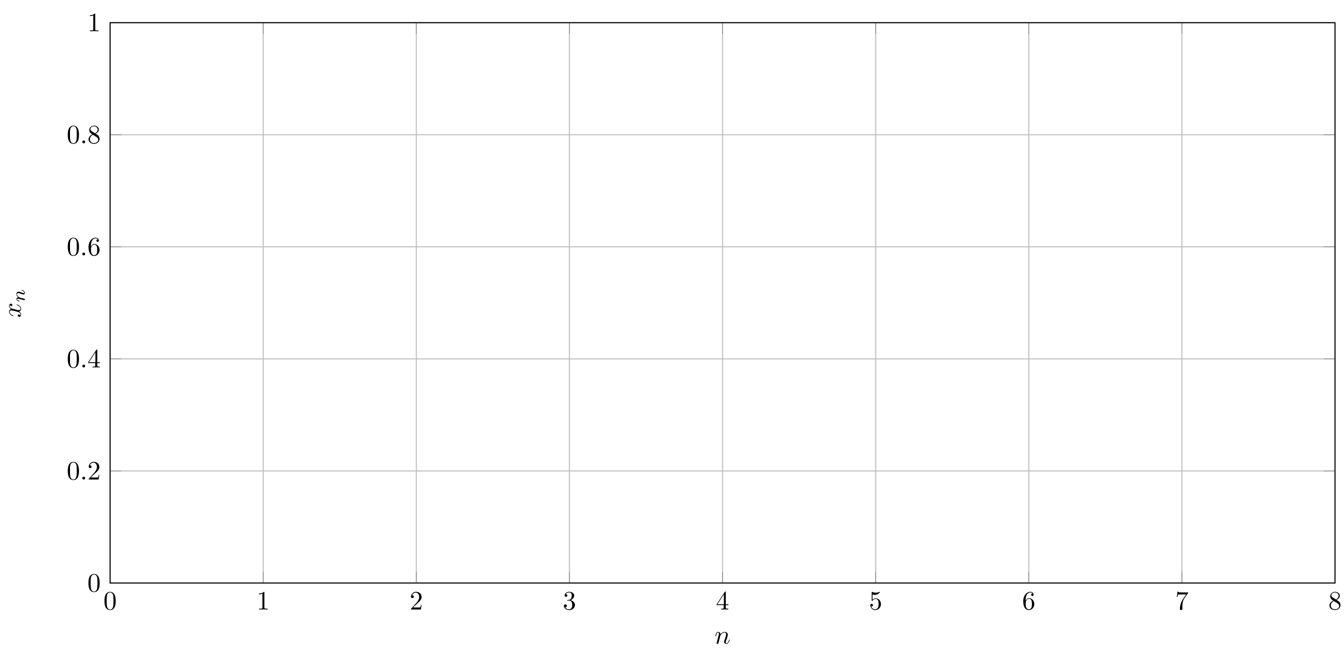



| | | |
|---|---|---|
| $x_0$ | | 0.3 |
| $x_1$ | $= f(x_0)$ | |
| $x_2$ | $= f(f(x_0))$ | |
| $x_3$ | $= f(f(f(x_0)))$ | |
| $x_4$ | $= f^4(x_0)$ | |
| $x_5$ | $= f^5(x_0)$ | |
| $x_6$ | $= f^6(x_0)$ | |

✍ How many period-$n$ orbits, if any, are there in this system, and at what value(s) of $x$ do they occur?

✍ What will happen to points initialized from values other than $x_0 = 0.3$ ?

✍ Plot $x_n$ versus $n$ on the following set of axes. What pattern do you notice?

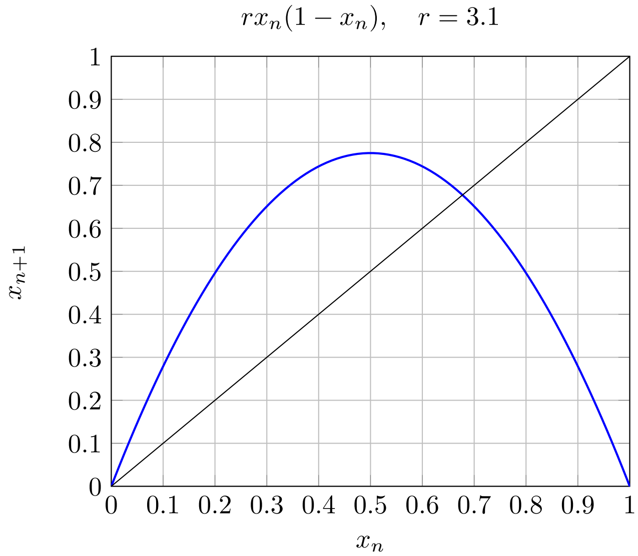


Now consider the map given by

$$x_{n+1} = r x_n (1 - x_n)$$
$$r = 3.1,$$

starting from $x_0 = 0.5$.

✍ Fill in the following table, and draw the cobweb diagram for this map.



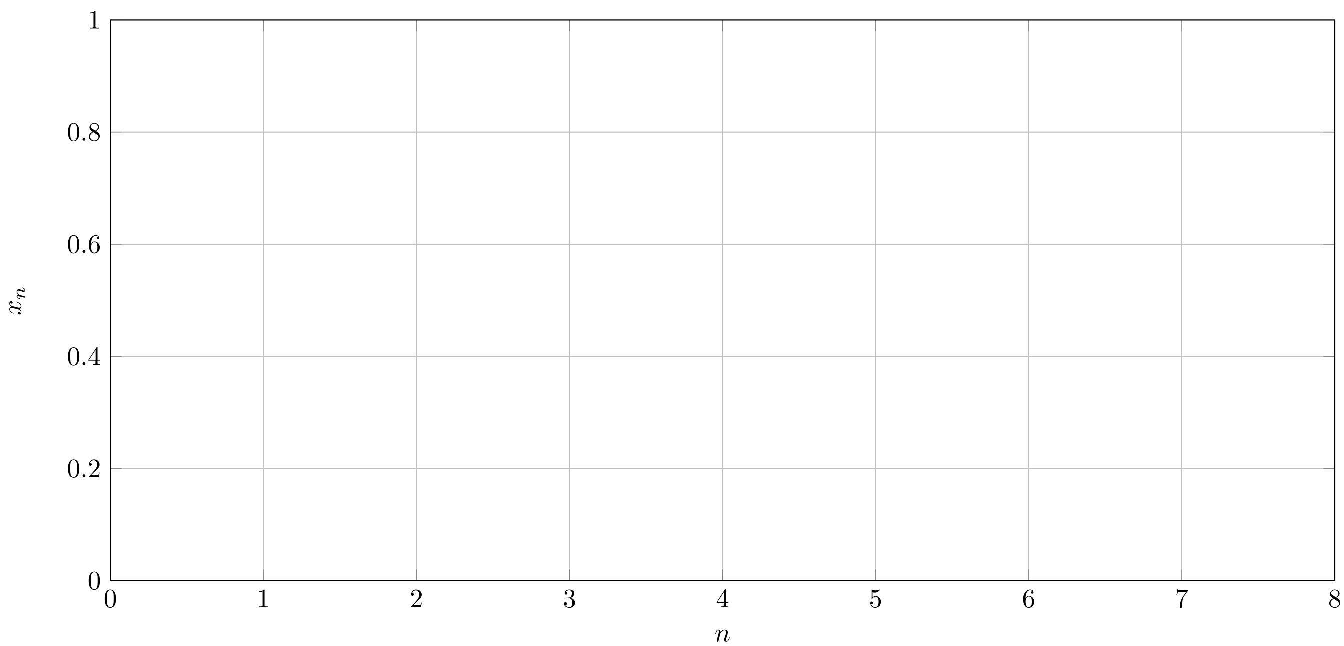

| | | |
|---|---|---|
| $x_0$ | | 0.5 |
| $x_1$ | $= f(x_0)$ | |
| $x_2$ | $= f(f(x_0))$ | |
| $x_3$ | $= f(f(f(x_0)))$ | |
| $x_4$ | $= f^4(x_0)$ | |
| $x_5$ | $= f^5(x_0)$ | |
| $x_6$ | $= f^6(x_0)$ | |
| $x_7$ | $= f^7(x_0)$ | |
| $x_8$ | $= f^8(x_0)$ | |

✍ How many period-$n$ orbits, if any, are there in this system, and at what value(s) of $x$ do they occur?

✍ Does the solution tend to a particular value ?

✍ What will happen for other initial conditions ?

✍ Plot $x_n$ versus $n$ on the following set of axes. What pattern do you notice?

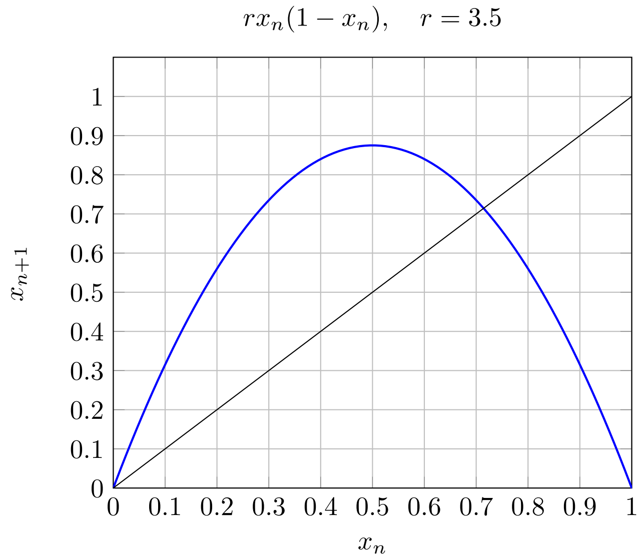


Now consider the map given by

$$x_{n+1} = r x_n(1 - x_n)$$
$$r = 3.5,$$

starting from $x_0 = 0.5$.

✍ Fill in the following table, and draw the cobweb diagram for this map.



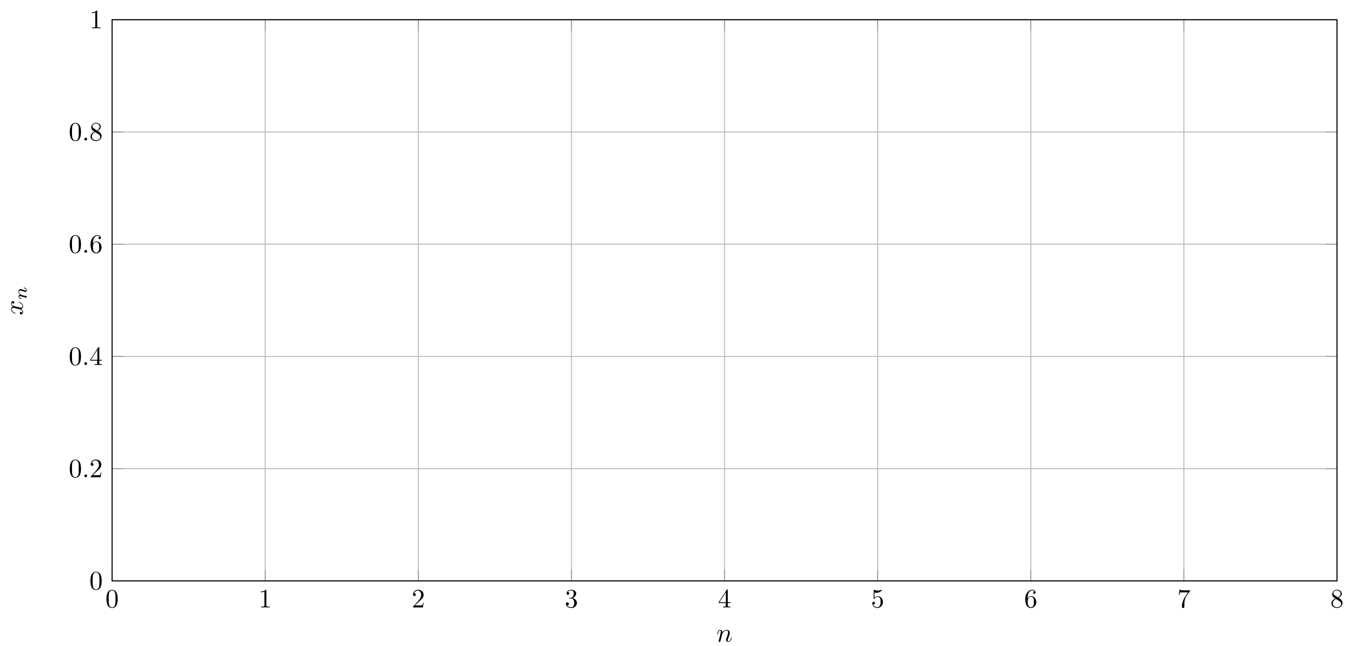

| | | |
|---|---|---|
| $x_0$ | | 0.5 |
| $x_1$ | $= f(x_0)$ | |
| $x_2$ | $= f(f(x_0))$ | |
| $x_3$ | $= f(f(f(x_0)))$ | |
| $x_4$ | $= f^4(x_0)$ | |
| $x_5$ | $= f^5(x_0)$ | |
| $x_6$ | $= f^6(x_0)$ | |
| $x_7$ | $= f^7(x_0)$ | |
| $x_8$ | $= f^8(x_0)$ | |

✍ Are there any period-$n$ orbits, in this system, and what is the value of $n$ ?

✍ Does the solution tend to a particular value ?

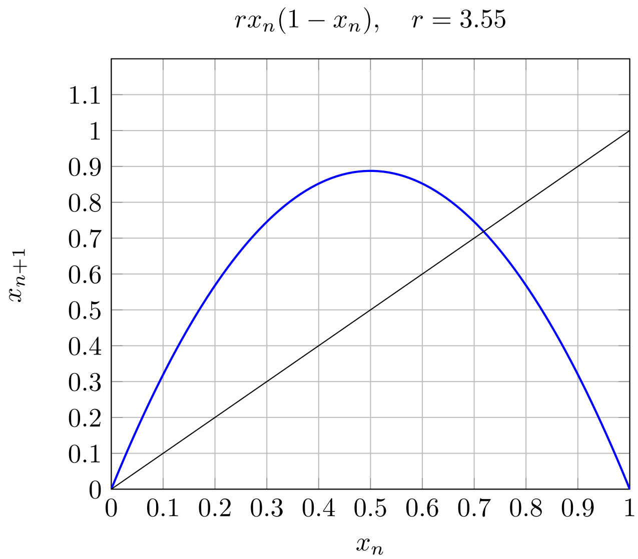
✍ What will happen for other initial conditions ?

✍ Plot $x_n$ versus $n$ on the following set of axes. What pattern do you notice?



Now consider the logistic map with $r = 3.55$. This time, you will iterate the map further.

✍ Fill in the following table, and draw the cobweb diagram for this map.



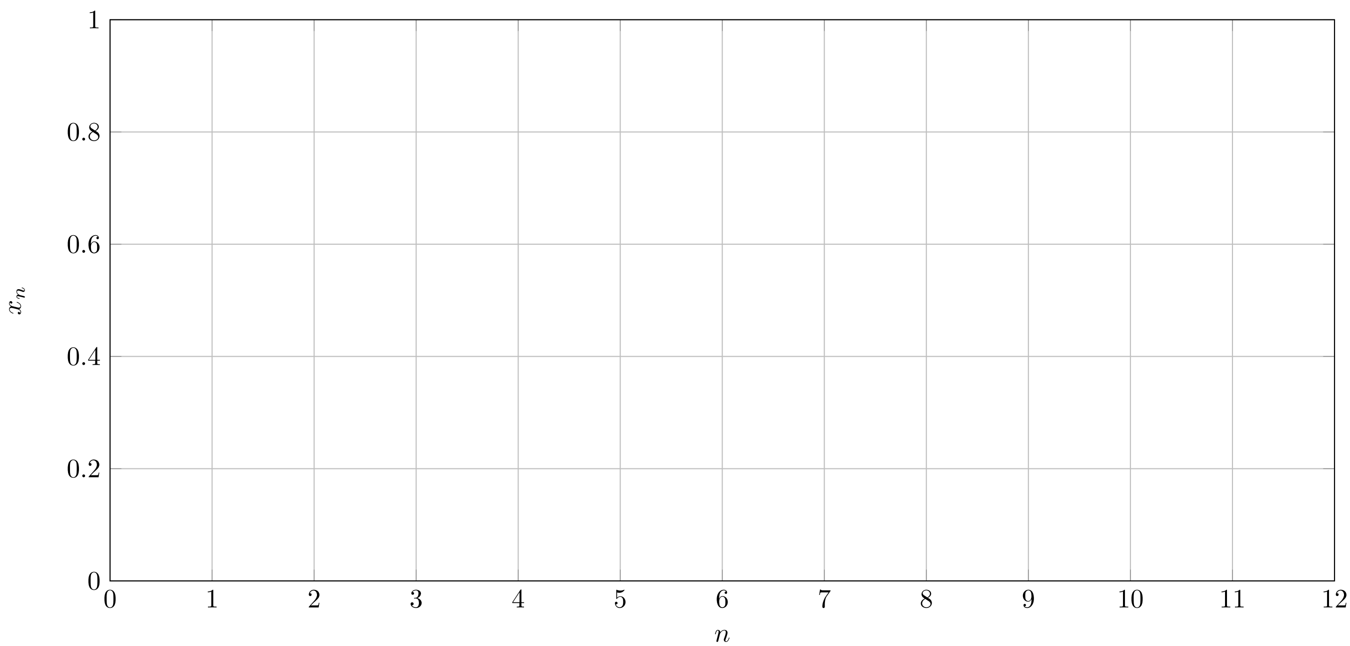

| | | |
|---|---|---|
| $x_0$ | | 0.5 |
| $x_1$ | $= f(x_0)$ | |
| $x_2$ | $= f(f(x_0))$ | |
| $x_3$ | $= f(f(f(x_0)))$ | |
| $x_4$ | $= f^4(x_0)$ | |
| $x_5$ | $= f^5(x_0)$ | |
| $x_6$ | $= f^6(x_0)$ | |
| $x_7$ | $= f^7(x_0)$ | |
| $x_8$ | $= f^8(x_0)$ | |
| $x_9$ | $= f^9(x_0)$ | |
| $x_{10}$ | $= f^{10}(x_0)$ | |
| $x_{11}$ | $= f^{11}(x_0)$ | |
| $x_{12}$ | $= f^{12}(x_0)$ | |

✍ Are there any period-$n$ orbits, in this system, and what is the value of $n$ ?

✍ Does the solution tend to a particular value ?

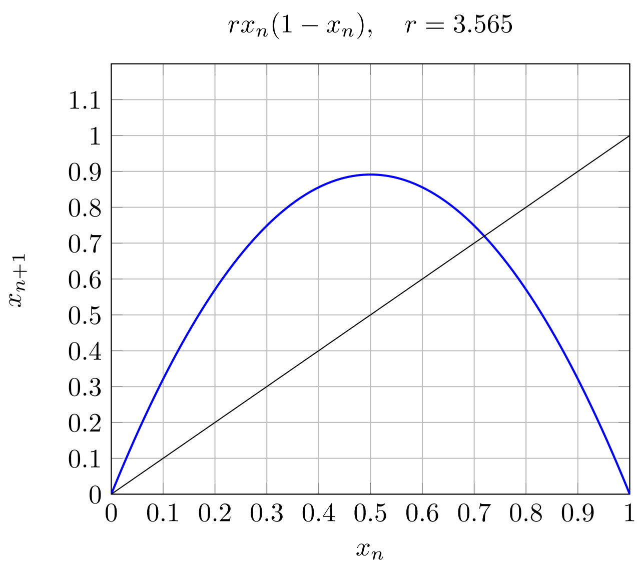
✍ What will happen for other initial conditions ?

✍ Plot $x_n$ versus $n$ on the following set of axes. What pattern do you notice?



Now consider the logistic map with $r = 3.565$. This time, you will iterate the map even further.

✍ Fill in the following table, and draw the cobweb diagram for this map.




| | | | | |
|---|---|---|---|---|
| $x_0$ | | 0.5 | $x_{13}$ | |
| $x_1$ | $= f(x_0)$ | | $x_{14}$ | |
| $x_2$ | $= f(f(x_0))$ | | $x_{15}$ | |
| $x_3$ | $= f(f(f(x_0)))$ | | $x_{16}$ | |
| $x_4$ | $= f^4(x_0)$ | | $x_{17}$ | |
| $x_5$ | $= f^5(x_0)$ | | $x_{18}$ | |
| $x_6$ | $= f^6(x_0)$ | | $x_{19}$ | |
| $x_7$ | $= f^7(x_0)$ | | $x_{20}$ | |
| $x_8$ | $= f^8(x_0)$ | | $x_{21}$ | |
| $x_9$ | $= f^9(x_0)$ | | $x_{22}$ | |
| $x_{10}$ | $= f^{10}(x_0)$ | | $x_{23}$ | |
| $x_{11}$ | $= f^{11}(x_0)$ | | $x_{24}$ | |
| $x_{12}$ | $= f^{12}(x_0)$ | | $x_{25}$ | |

✍ Are there any period-$n$ orbits, in this system, and what is the value of $n$ ?

✍ Does the solution tend to a particular value ?

✍ What will happen for other initial conditions ?

✍ Plot $x_n$ versus $n$ on the following set of axes. What pattern do you notice?