

1. The Lorenz equations. Consider the system of ordinary differential equations studied by Edward Lorenz as a model for atmospheric convection.

$$\dot{x} = \sigma(y - x) \quad (1a)$$

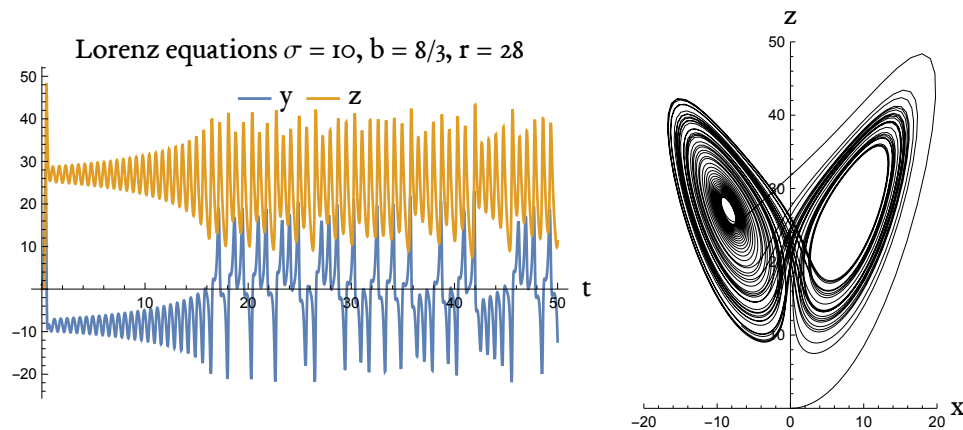
$$\dot{y} = rx - y - xz \quad (1b)$$

$$\dot{z} = xy - bz, \quad (1c)$$

where $\sigma, r, b > 0$. For each of the following parts, you should produce two plots:

1. (Time series) Plot $y(t)$ and $z(t)$ on the same set of axes, with t on the horizontal axis.
2. (Parametric) Plot $x(t)$ against $z(t)$, i.e., place x on the vertical axis and z on the horizontal axis. The parametric plot should have the **same** axis limits across all plots.

An example of the kinds of plots you are expected to produce is shown below.



- (a) Numerically solve (1) using a programming language of your choice for the following parameters

- $\sigma = 10, b = 8/3, r = 21.5$
- $x(0) = 0, y(0) = 1, z(0) = 20$.
- integrate from $t = 0$ to $t = 100$,

and plot the results. Describe this behaviour in words. Do you see any fixed points, limit cycles, or chaos?

- (b) Numerically solve (1) using a programming language of your choice for the following parameters

- $\sigma = 10, b = 8/3, r = 14$
- $x(0) = 0, y(0) = 1, z(0) = 0$.
- integrate from $t = 0$ to $t = 20$,

and plot the results. Describe this behaviour in words. Do you see any fixed points, limit cycles, or chaos?