

Mon, Apr 7 Lecture 19

Time Horizon of Prediction in systems with sensitive dependence on initial conditions.

$$\|\delta(t)\| \sim \|\delta_0\| e^{\lambda t}$$

for Lorenz system
 $\lambda \approx 0.9$ when

$$\sigma=10, b=\frac{8}{3}, r=28$$

Find time t^* at which two initially nearby (δ_0) trajectories have diverged by more than ϵ .

$$\epsilon \approx \|\delta_0\| e^{\lambda t^*}$$

$$\Rightarrow \frac{\epsilon}{\|\delta_0\|} \approx e^{\lambda t^*}$$

\Rightarrow

$$t^* \approx \frac{1}{\lambda} \log \left[\frac{\epsilon}{\|\delta_0\|} \right]$$

Example

Two measurements were made to a precision of $\delta_0 = 10^{-7}$

We consider deviations more than $\epsilon = 10^{-3}$ to be unacceptable.

Increase initial precision.

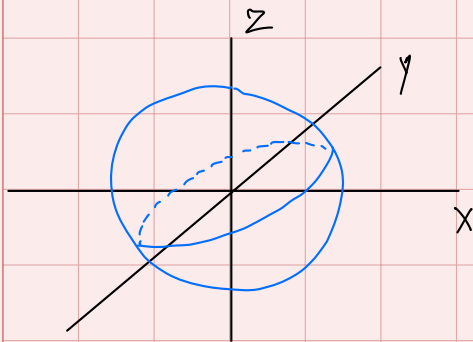
Now, $\delta_0 = 10^{-15}$ (100,000,000 x more precise)

$$\Rightarrow t^* \approx 10.2 \quad \begin{matrix} \nearrow 3x \end{matrix}$$

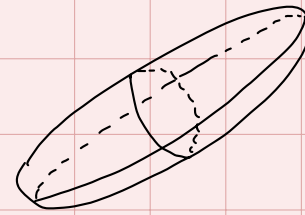
$$\Rightarrow t^* \approx 30.7$$

the largest Liapunov Exponent of the system. ≈ 0.9 for Lorenz

What exactly is λ ?



time



sphere of initial conditions
with infinitesimal radius δ

Ellipsoid with principal
axes

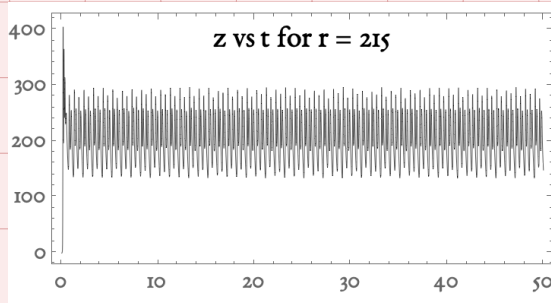
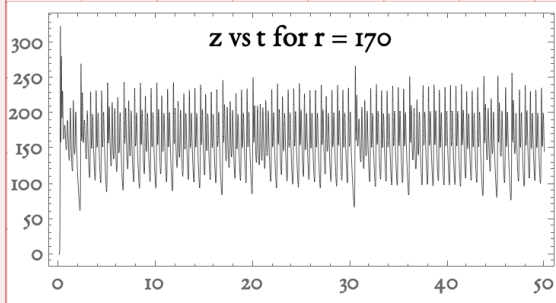
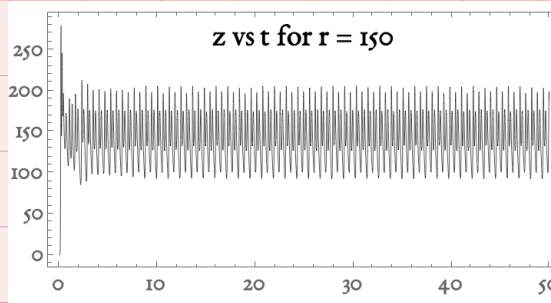
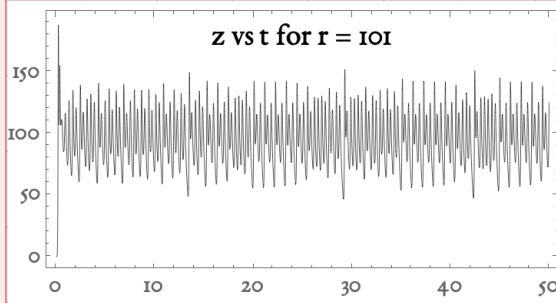
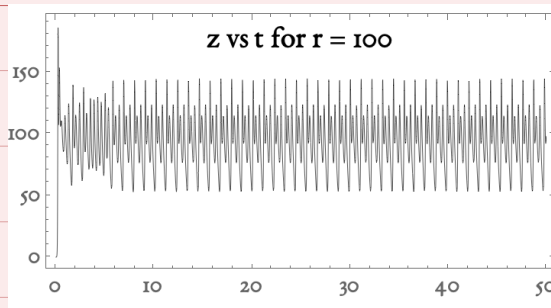
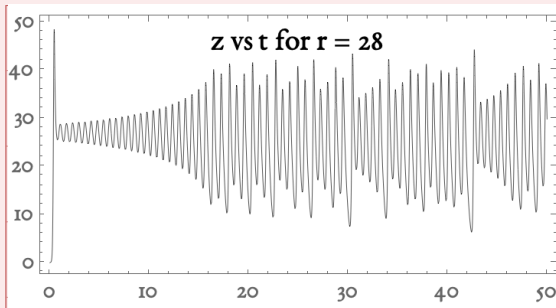
$$\begin{array}{rcl}
 \delta_1(0) & \longrightarrow & \delta_1(0)e^{\lambda_1 t} \\
 \delta_2(0) & \longrightarrow & \delta_2(0)e^{\lambda_2 t} \\
 \delta_3(0) & \longrightarrow & \delta_3(0)e^{\lambda_3 t}
 \end{array}$$

At large times, the largest λ_k controls the size of the ellipsoid.
Liapunov Exponent.

Negative $\lambda_k \Rightarrow$ that direction shrinks
Positive $\lambda_k \Rightarrow$ that direction expands.

If Liapunov Exponent > 0 , \Rightarrow sensitive dependence on initial conditions.

tinyurl.com/E91lorenz4 \longrightarrow remix



order \longrightarrow chaos \longrightarrow order \longrightarrow chaos \longrightarrow ?