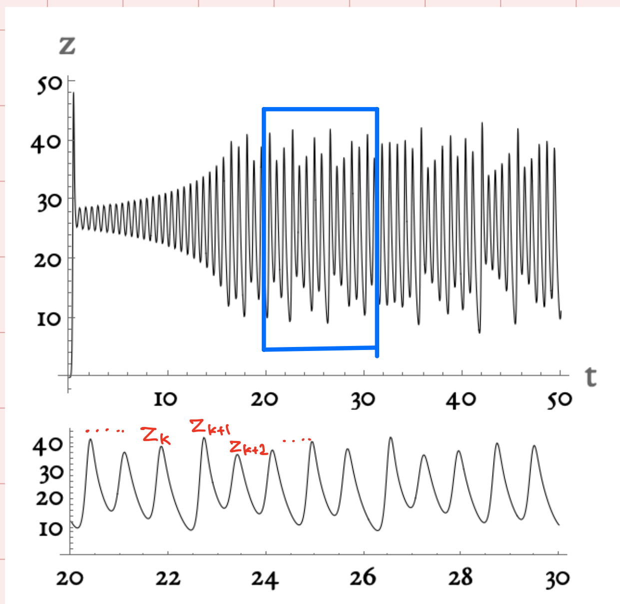
Wed, Apr 9 Lecture 20



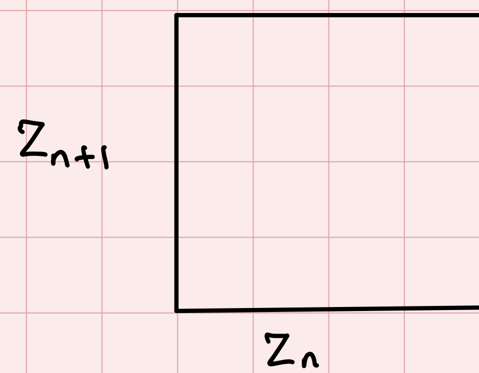
Continuous Time → Discrete Time

Differential Eqs → Maps

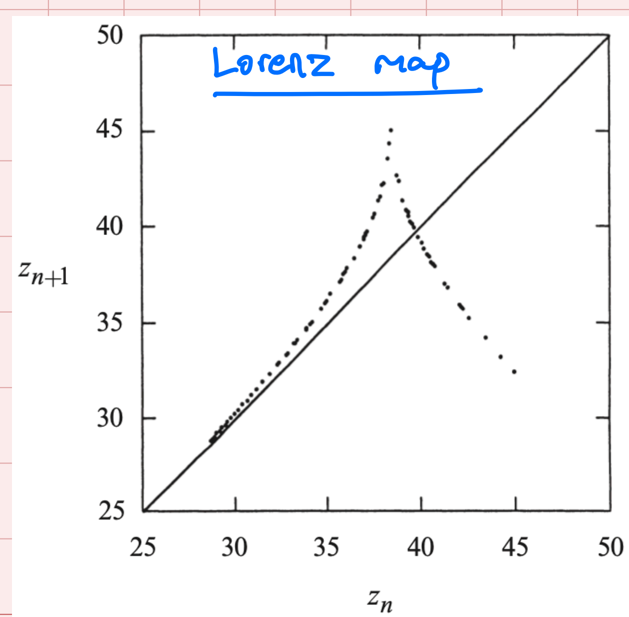
$\dot{x} = f(x)$ $x_{n+1} = f(x_n)$

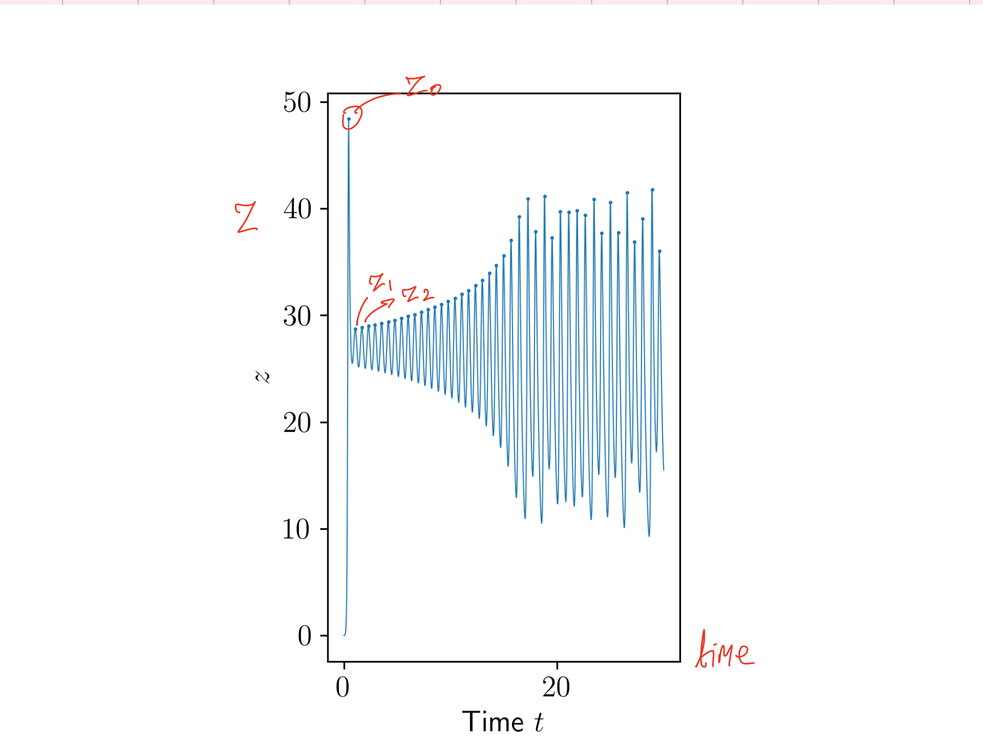
These two are not the same, even if you have a "map" and a differential eqn. describing the same underlying system.

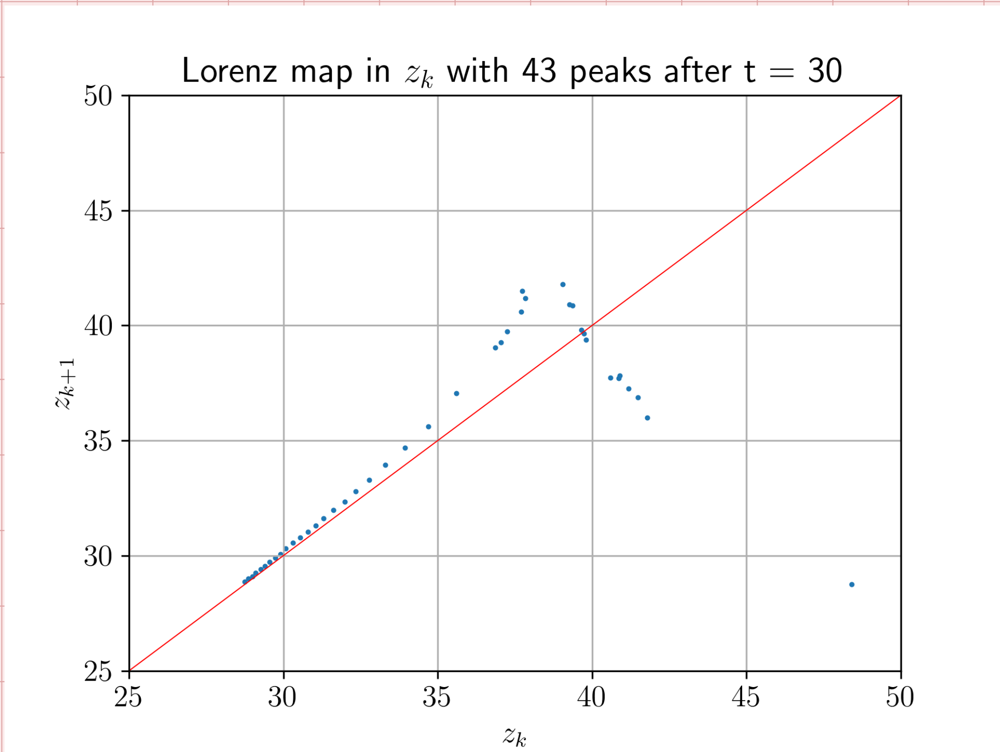
If you know "f" for a map, you can plot → by graphing the function.

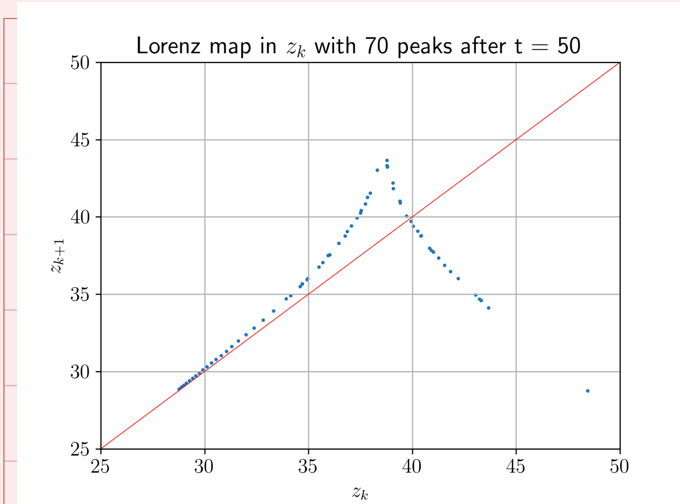


For Lorenz system, you have to do this empirically



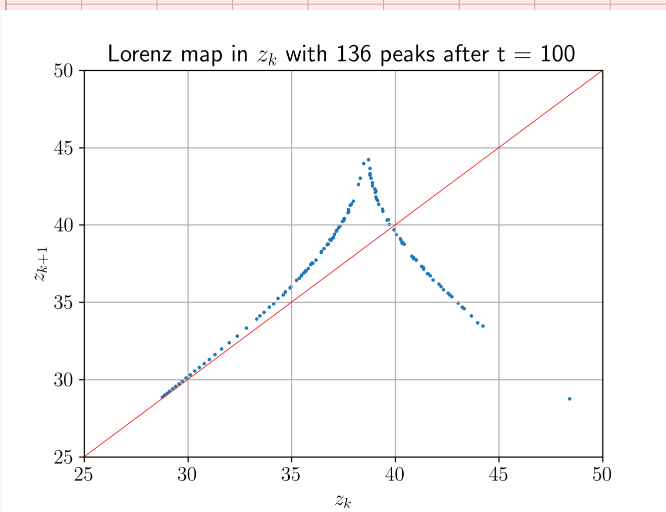

Z_0
Z
Z_1
Z_2
time
z
Time t


Lorenz map in z_k with 43 peaks after t = 30
z_{k+1}
z_k

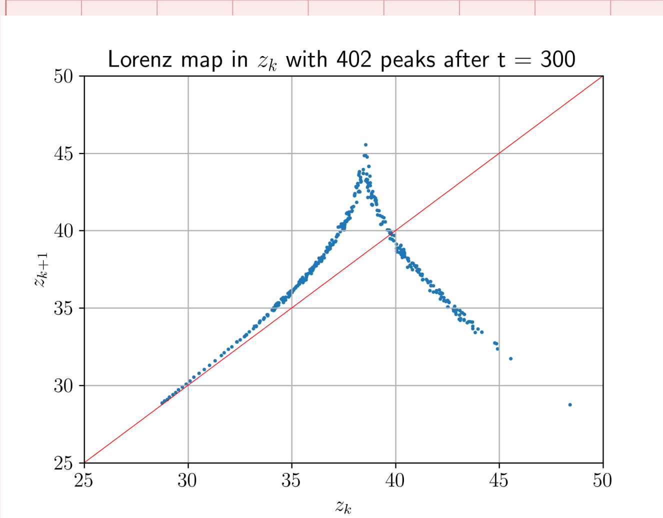


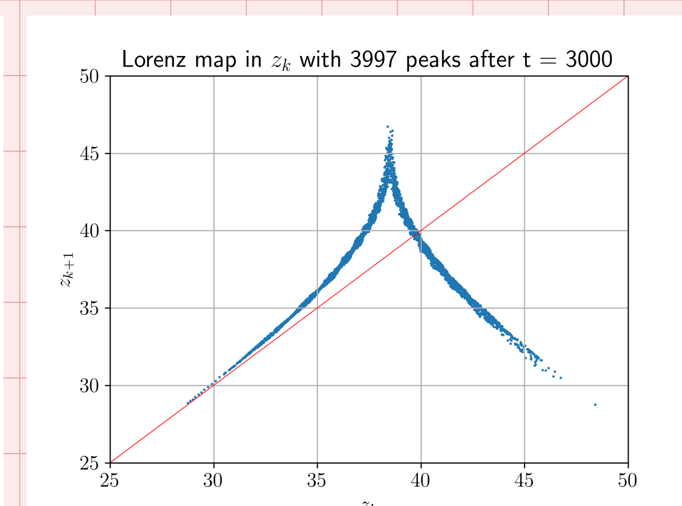
All figures produced with $\sigma = 10$, $b = 8/3$, $r = 28$

when Lorenz system is known to be chaotic.



looks like graph of $1 - \sqrt{|2z - 1|}$ with appropriate shifts and rescaling.





Why are we looking at maps?

→ Behaviour of Lorenz system is not well-understood. A simple(r) mathematical model that captures some aspects of the Lorenz system might give us some insight.

→ Can 1-d maps have chaos?

→ In some physical systems, it makes sense to think of time as discrete

$$x_{n+1} = f(x_n) \qquad f: \mathbb{R} \to \mathbb{R}$$

- f is a "map" from $\mathbb{R}$ to $\mathbb{R}$
- We'll study f's that are continuous and piecewise smooth. (no jumps ; cusps/kinks are allowed)

Some more examples of maps