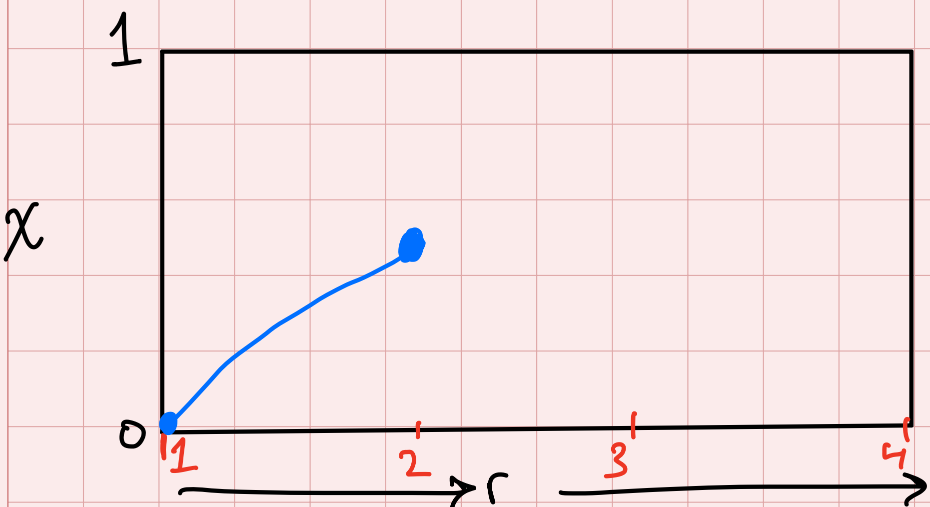
Wed, Apr 16 Lecture 22

As r increases, the period of the cycle doubles. (Successively)

$\Rightarrow$ Draw the orbit diagram (of the Logistic map)



similar to a bifurcation diagram but only includes stable structures. → orbits : $\underbrace{\text{period-1 orbits}}_{\text{fixed pts}}$, period-2 orbits, ...

At $r = 1$, how does logistic map behave?

Look for solutions to $\{ x_{n+1} = x_n(1-x_n)\cdot r \ , \ x_n = x_{n+1} \}$

$x = x(1-x)$

$x = x - x^2 \Rightarrow x = 0$ only solution.

At $r=2$: Solve $x = \overbrace{2}^{r=2}x(1-x)$

$x = 2x - 2x^2$

$2x^2 - x = 0$

$x(x - \frac{1}{2}) = 0$ → $x = \frac{1}{2}$ happens to be stable

two solutions! (ignore $x = 0$; unstable)

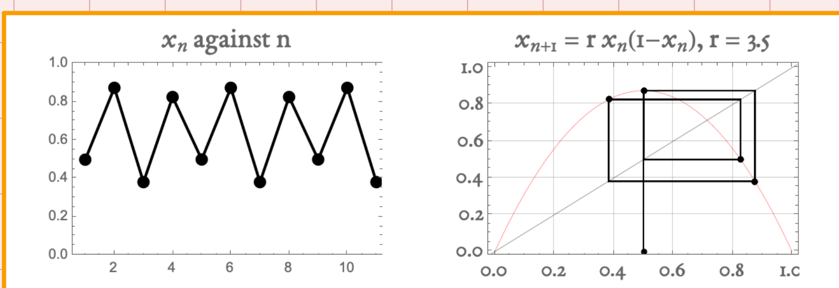
At $r = 3.5$: ... $x_{n+1} = \frac{7}{2} x_n (1 - x_n)$, $x_{n+1} = x_n$

$\vdots$

$x(x - 5/7) = 0$

$x = 5/7$ is a fixed pt.

↓



unstable, doesn't make it into the orbit diagram.

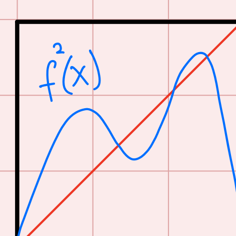
Fixed points of the 2^{nd} iterate map correspond to period-2 orbits.

Example

i.e find x_n such that $x_{n+2} = x_n$ for some map f.

$f(f(x_n)) = x_n$

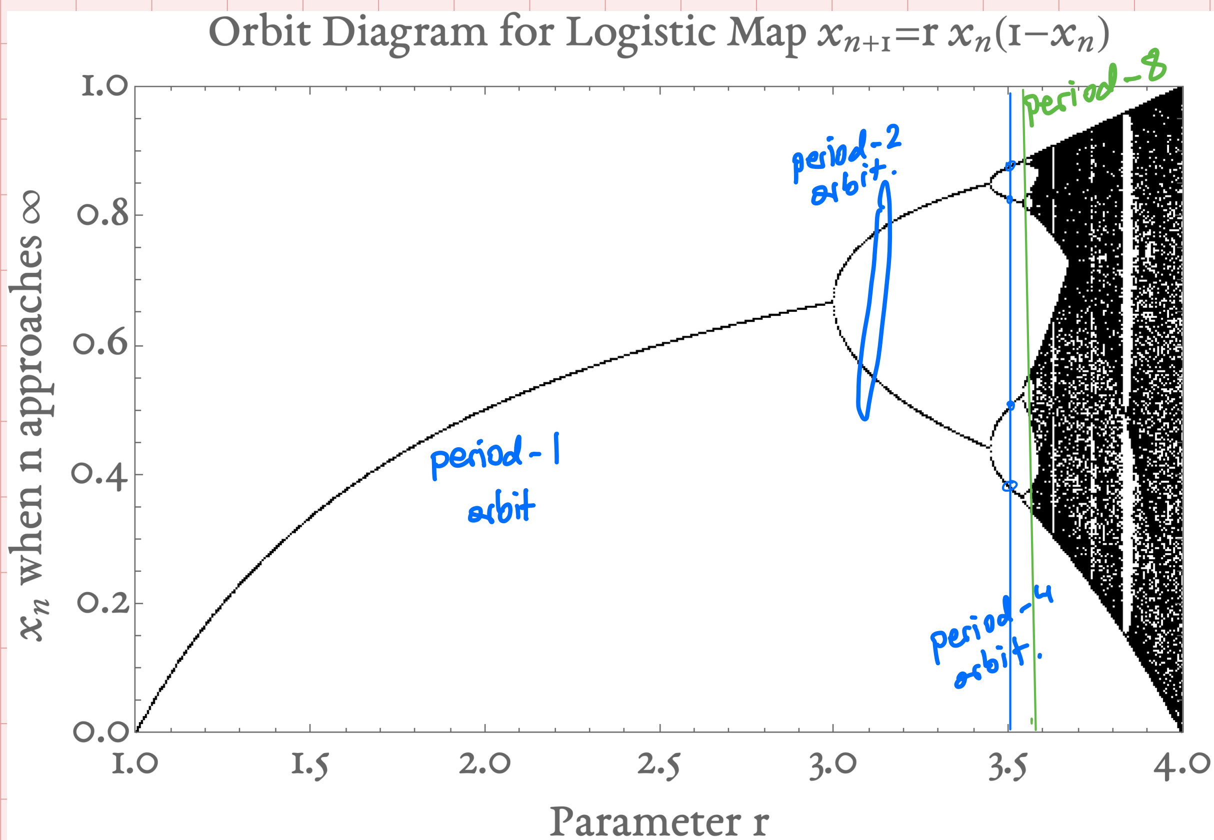
⇒ a period-k orbit is solution of $x_{n+k} = x_n$, $f^k(x) = x$.



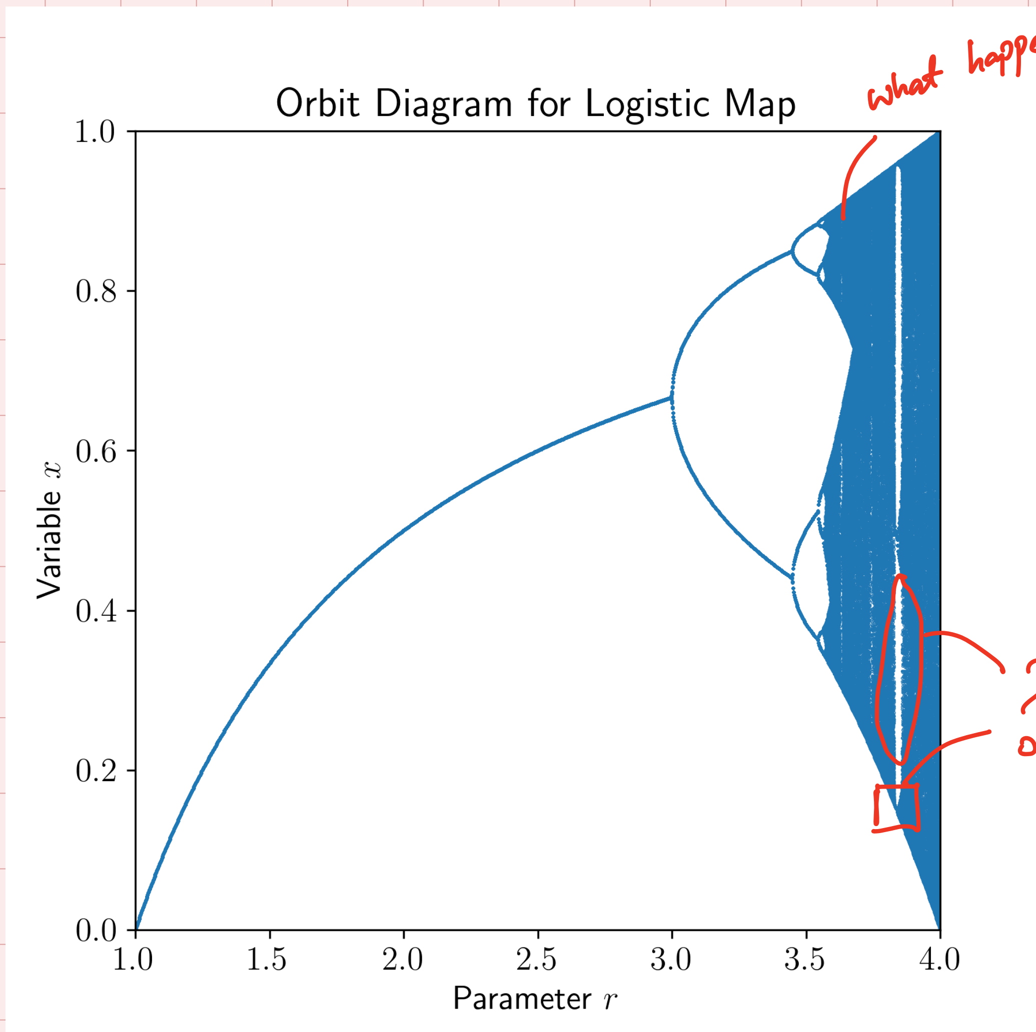
Typically, graphical methods are used.

Period-doubling route to chaos

At large r, between ~3.7 and 4, the logistic map shows signs of chaos.



- Pick r.
- we start out with <u>any</u> value of x
- Iterate the map, say, 1200 times. $x_0 \to x_1 \to x_2$
- Drop the first, say, 1000 values.
- Plot <u>all</u> of the most recent 200 values.



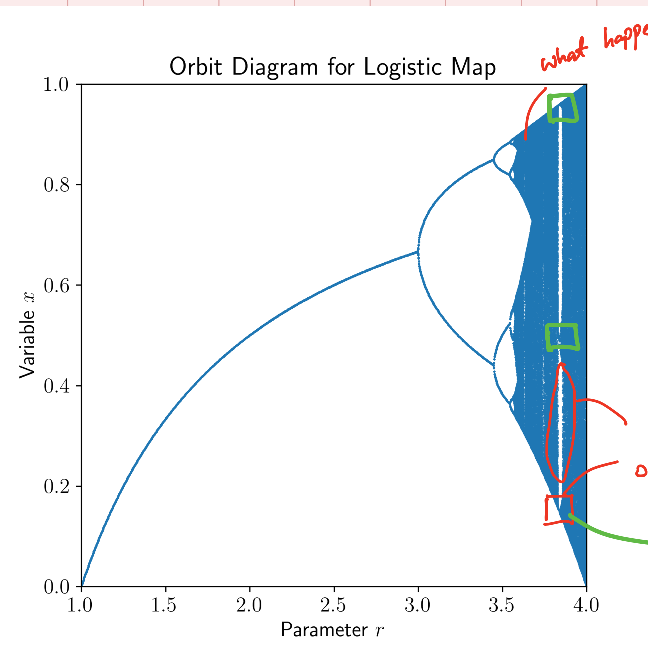
what happens here?

"period-∞ orbit"

??

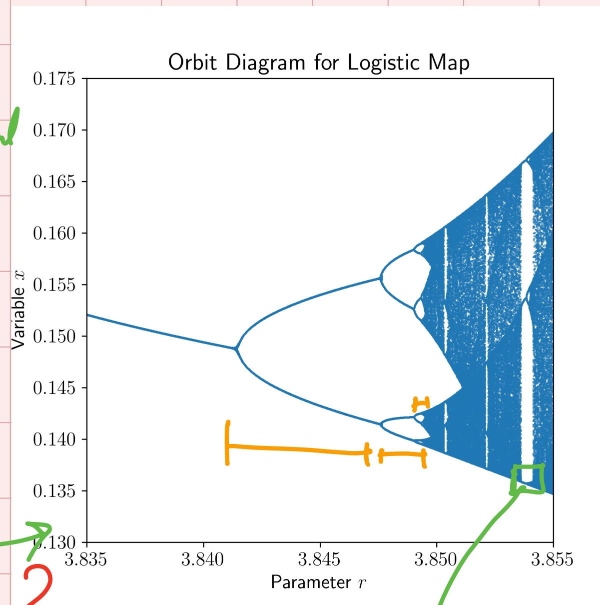
- A period-k orbit comes back after k steps.
- A period-∞ orbit $\underbrace{\text{comes back after } \infty \text{ steps.}}_{\text{never repeats itself exactly}}$

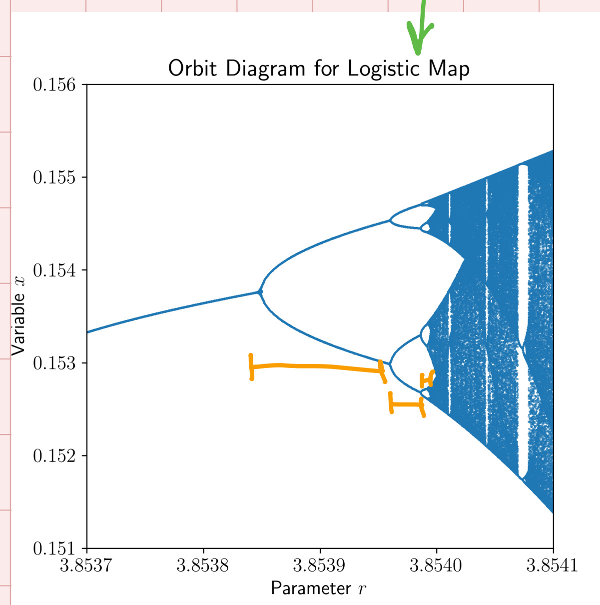
At some $\underbrace{\text{limiting value of } r}_{\text{a finite irrational number}}$, $k \longrightarrow \infty$

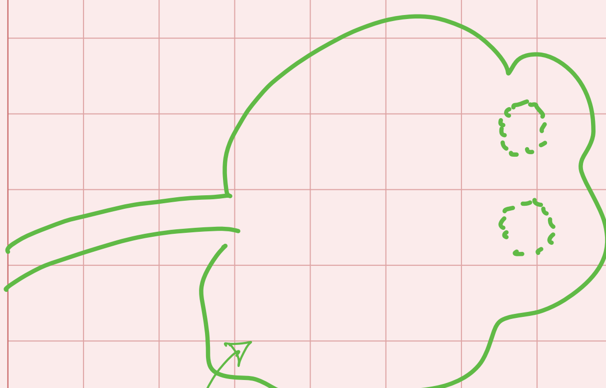


what happen

"copies" of original orbit diagram







This is a Broccoli