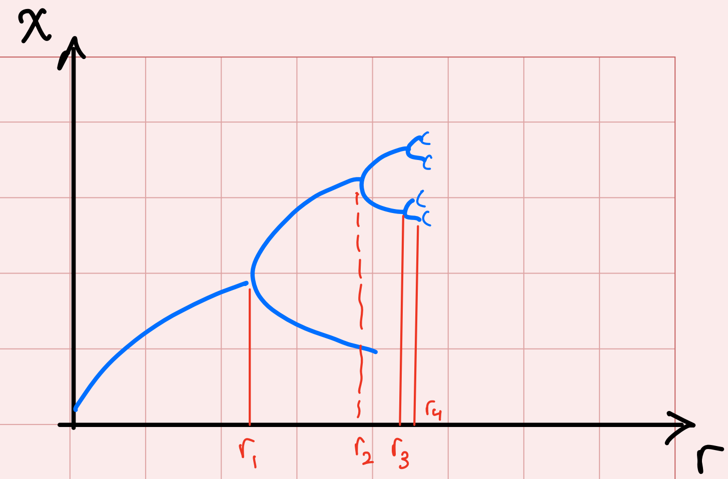
Mon, Apr 21 Lecture 23

Features of Orbit Diagram

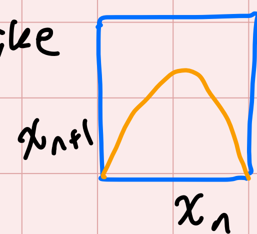


$$\boxed{\delta = \lim_{n\to\infty} \frac{r_n - r_{n-1}}{r_{n+1} - r_n} \approx 4.669}$$

Feigenbaum's constant δ

characterizes period-doubling in all unimodal maps

i.e. anything with a graph like

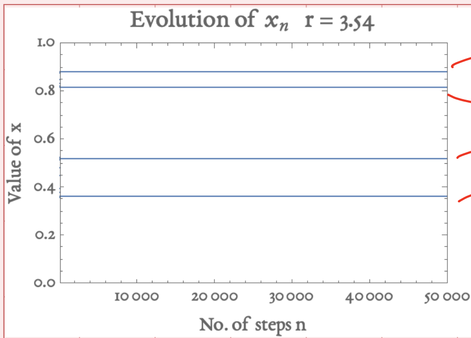


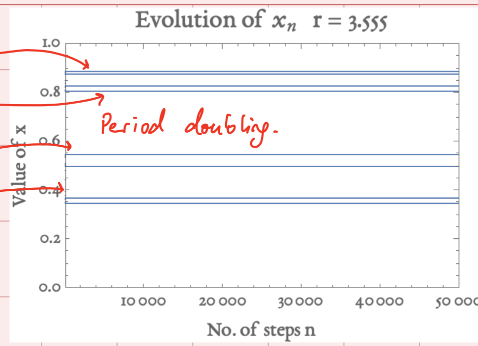
For some value of r, $3.55 < r < 3.70$,

the system becomes chaotic : orbit diagram is

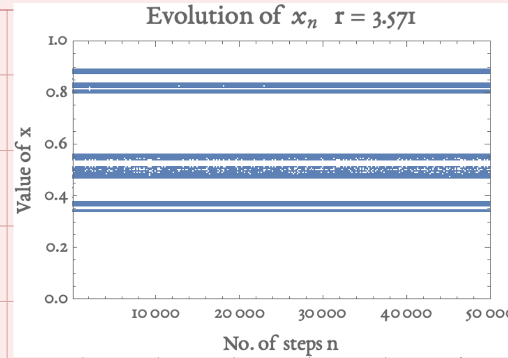
dense: no value is ever repeated, but you basically

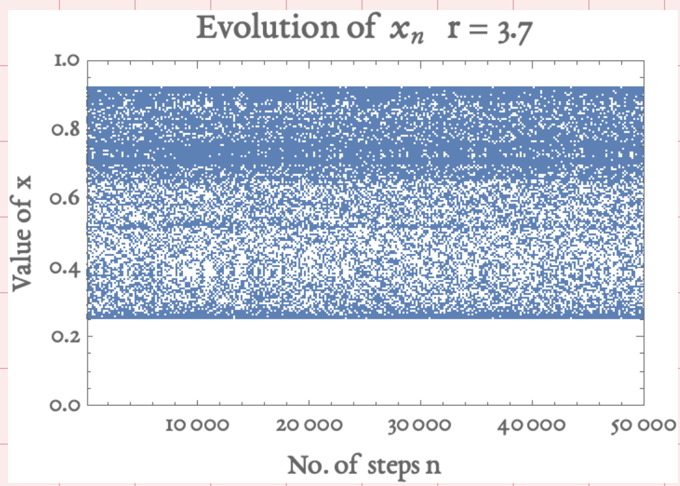
cover "all numbers" b/w 0 and 1



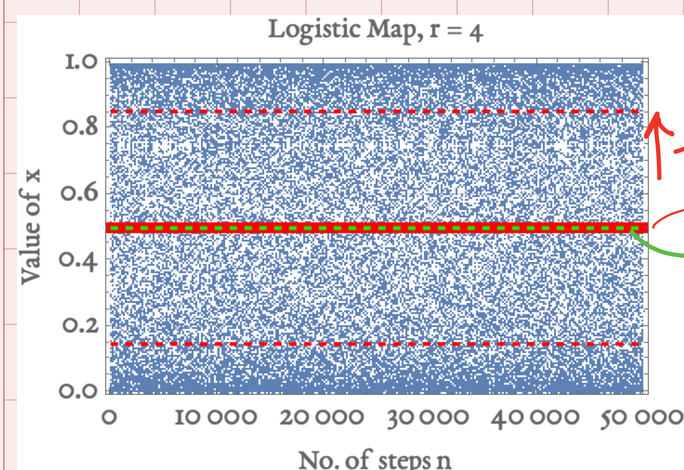


Period doubling.





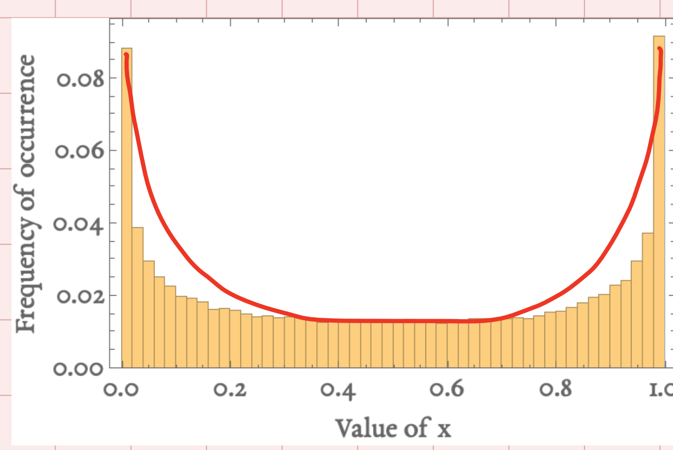
$$x_{n+1} = r x_n (1 - x_n)$$



one standard deviation

Mean

median



[PDF]

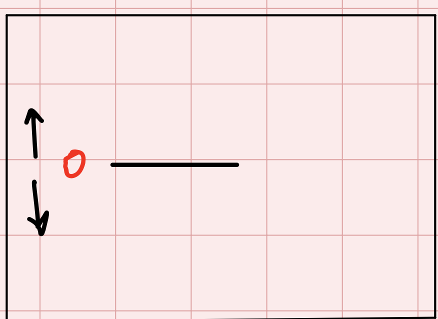
The probability density function that emerges from this data — shaped like ⌣ — is reminiscent of the probability distribution known as arcsine:

A distribution whose cumulative distribution (CDF) is $\frac{2}{\pi}\arcsin\sqrt{x}$ and its PDF is $\frac{1}{\pi\sqrt{x(1-x)}}$

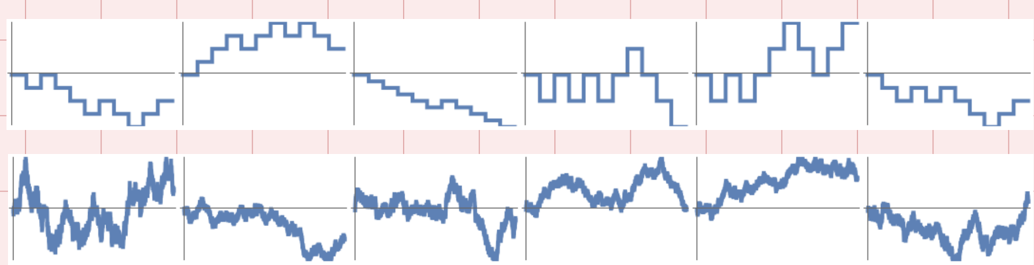
~ related to random walks

each step : two possibilities : $\left.\begin{array}{l}\text{up } (+1)\\ \text{down } (-1)\end{array}\right\}$ take n steps.

$\underbrace{}_{\text{probability } 1/2}$



over time, is it more likely that he will spend more time on one side of the line or is it more likely that he will spend equal amounts of time above & below?

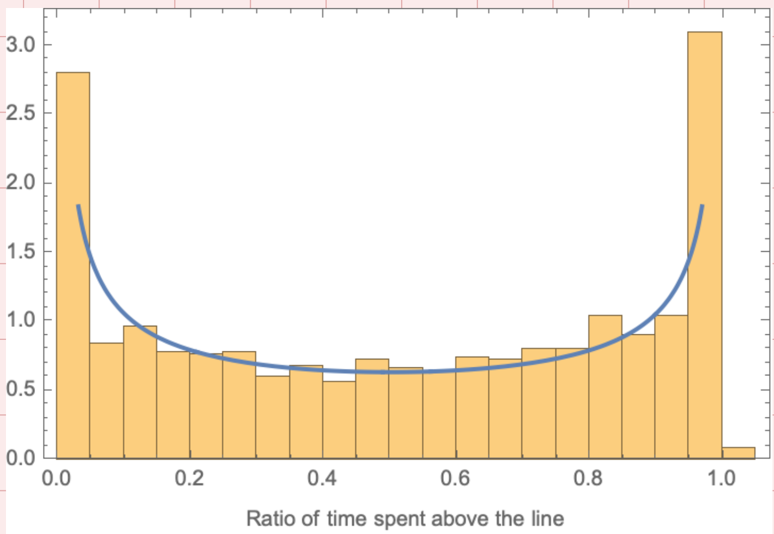


10 steps

10,000 steps

cumulative area under curve is the probability that one person will win a long coin-toss game. If area is zero, a tie will occur. ⇒ Ties are unlikely.

This process leads to an arcsine distribution of outcomes



When a large number of coin-toss games are sampled, the proportion of 'time' for which one player is in the lead is very high, and a long-running tie is unlikely.

Player 1 winning by large margin

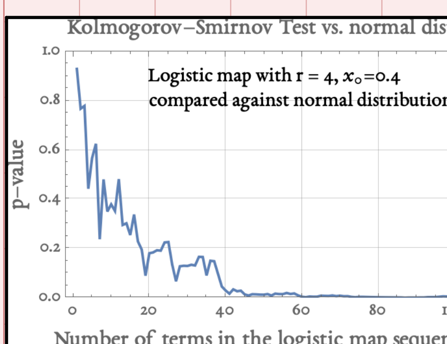
Tie

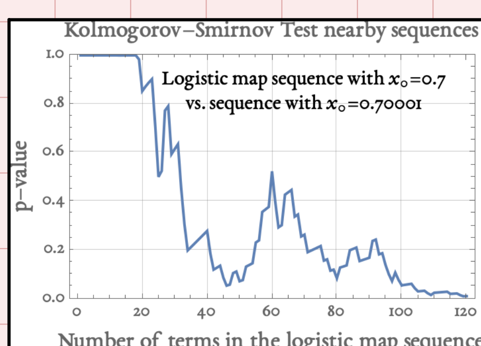
Player 2 winning by large margin

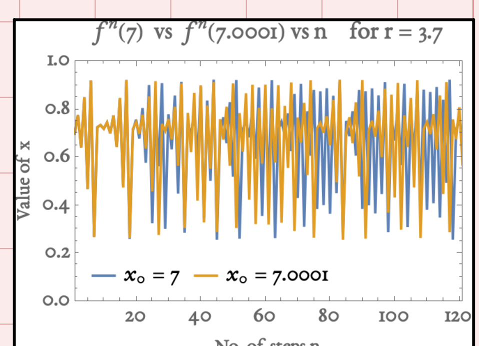
Kolmogorov-Smirnov Test

Compare a sequence $\{x_0\ x_1\ x_2 \dots x_n\}$ against:-

1) a normal distribution
2) a different sequence with slightly different x_0.







p-value: probability that data came from tested distribution

Autocorrelation Test

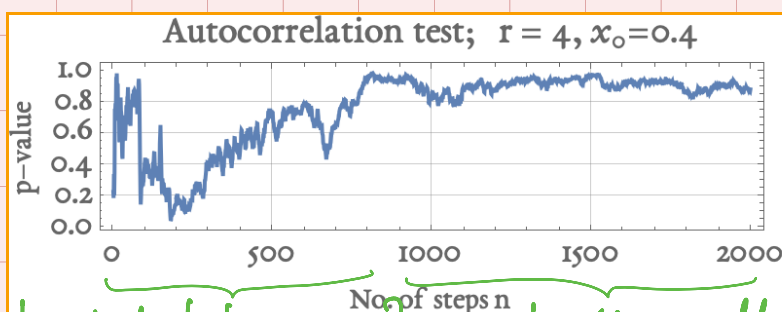
Is a signal correlated with itself? Noise is not correlated with anything, not even itself.

$$\{a, b, c, d, \dots \dots, x, y, z, \dots\}$$

shift left.

$$\{c, d, e, \dots \dots, x, y, z, \dots \dots\}$$

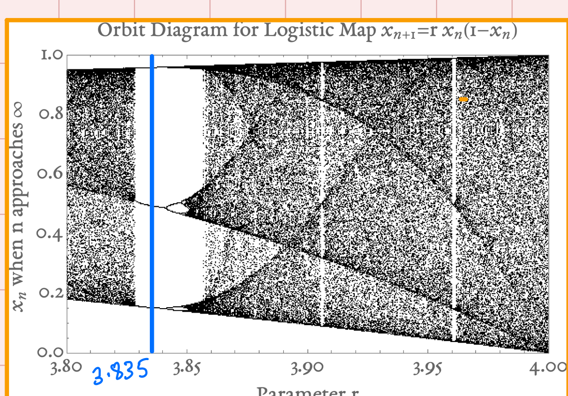
compare: are they close? if yes, $\Rightarrow$ autocorrelated.



transient behaviour? chaotic attractor?

Existence of "periodic windows" can be predicted by examining the n^{th} iterate map $f^n(x)$

— : $f(x)$ — $f^2(x)$ — $f^3(x)$



period-3 orbit