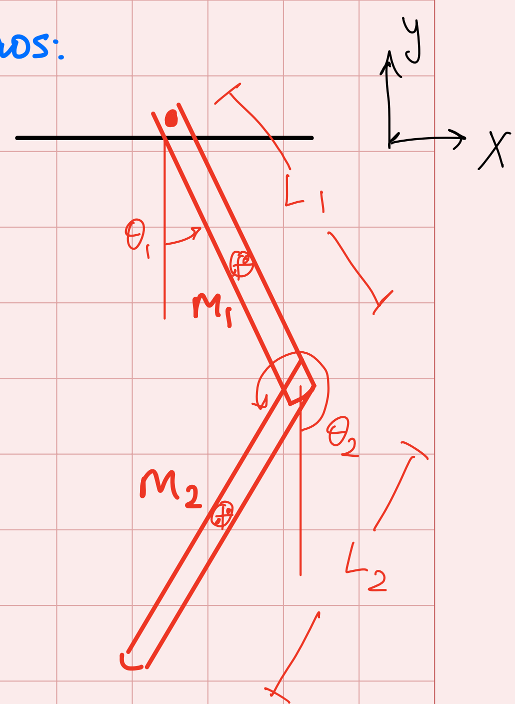
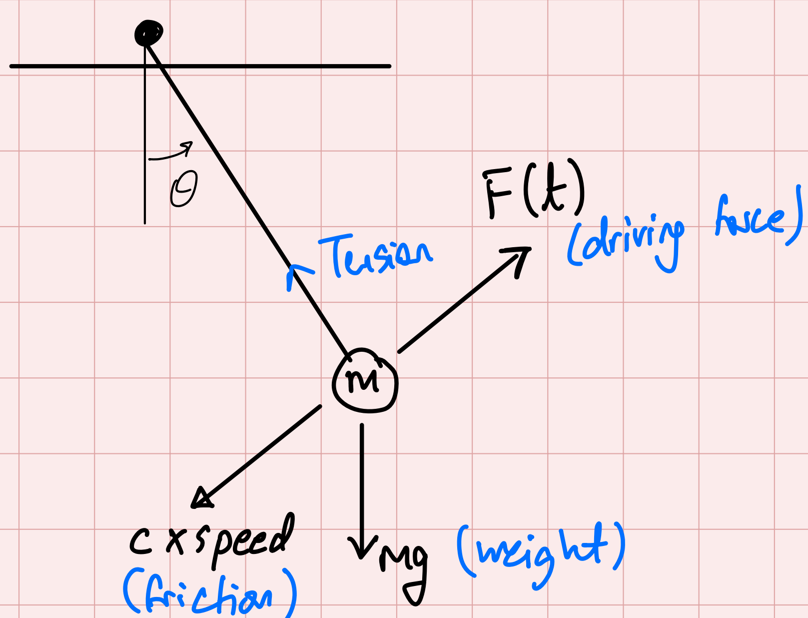
Wed, Apr 23 Lecture 24

Physical Systems that exhibit Chaos:

1) The double $\underbrace{\text{(compound)}}_{\text{distributed mass}}$ pendulum with or without damping.



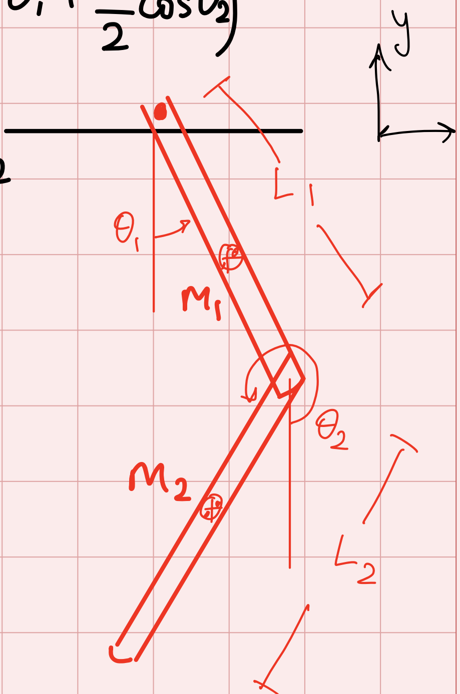
2) The simple driven pendulum (with damping)



① Potential Energy $V = mgy_1 + mgy_2$

$$= -m_1 g \frac{L_1}{2}\cos\theta_1 - m_2 g\left(L_1\cos\theta_1 + \frac{L_2}{2}\cos\theta_2\right)$$

Kinetic Energy $T = \frac{1}{2} m_1 \vec{v}_1 \cdot \vec{v}_1 + \frac{1}{2} m_2 \vec{v}_2 \cdot \vec{v}_2$



$$\vec{r}_1 = \begin{bmatrix} +\frac{L_1}{2}\sin\theta_1 \\ -\frac{L_1}{2}\cos\theta_1 \end{bmatrix}$$

$$\vec{r}_2 = \begin{bmatrix} +L_1\sin\theta_1 + \frac{L_2}{2}\sin\theta_2 \\ -L_1\cos\theta_1 - \frac{L_2}{2}\cos\theta_2 \end{bmatrix}$$

$$\vec{v} = \frac{d}{dt}(\vec{r}) \Rightarrow \vec{v}_1 = \begin{bmatrix} \frac{L_1}{2}\dot{\theta}_1\cos\theta_1 \\ \frac{L_1}{2}\dot{\theta}_1\sin\theta_1 \end{bmatrix}$$

similarly for $\vec{v}_2$.

$$\therefore \; T = \underbrace{\frac{L_1^2 m_1}{8}\dot{\theta}_1^2}_{\text{first rod}} + \underbrace{\frac{1}{2}m_2 L_1^2\dot{\theta}_1^2 + \frac{1}{2}m_1 L_1 L_2\cos(\theta_1 - \theta_2)\dot{\theta}_1\dot{\theta}_2 + \frac{1}{8}m_2 L_2^2\dot{\theta}_2^2}_{2^{nd}\text{ rod}}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_j}\right) - \frac{\partial L}{\partial \theta_j} = \begin{Bmatrix} \text{non-conservative} \\ \text{force terms} \end{Bmatrix}$$

e.g. "$-c\dot{\theta}$"

$j = \{1, 2\}$

Two nonlinear 2^{nd} order non-autonomous differential eq.

- each equation can have $\{\ddot{\theta}_1\ \dot{\theta}_1\ \theta_1\ \ddot{\theta}_2\ \dot{\theta}_2\ \theta_2\}$
- coupled — can't solve separately.
- can convert to four 1^{st} order ODEs

unknowns: $\begin{Bmatrix} \dot{\theta}_1(t) & \dot{\theta}_2(t) \\ \theta_1(t) & \theta_2(t) \end{Bmatrix} \longrightarrow$ call them x_i

$$\dot{\vec{x}} = \vec{f}(\vec{x})$$

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1\ x_2\ x_3\ x_4) \\
\dot{x}_2 &= f_2(x_1\ x_2\ x_3\ x_4) \\
\dot{x}_3 &= f_3(x_1\ x_2\ x_3\ x_4) \\
\dot{x}_4 &= f_4(x_1\ x_2\ x_3\ x_4)
\end{aligned}$$

- Phase space is 4-dimensional.
- if you use small-angle approximation (carefully) then $\vec{f}(\vec{x})$ becomes $A\vec{x}$