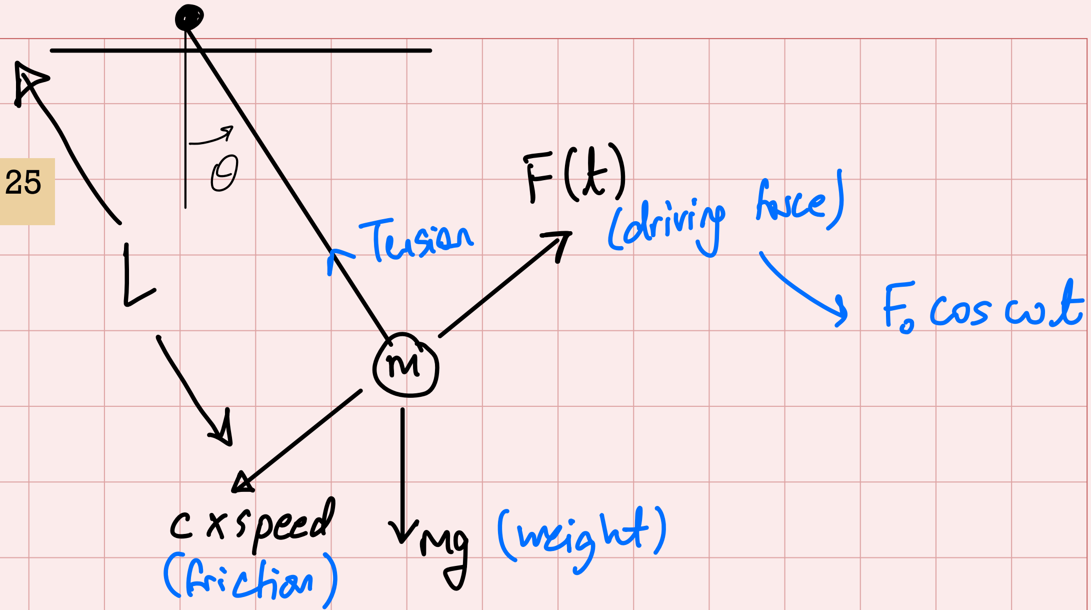
Mon, Apr 28 Lecture 25



2nd order nonautonomous

$$\underbrace{I}\,\ddot{\theta} = \sum \text{torques}$$

$$mL^2\ddot{\theta} = -cL^2\dot{\theta} - mgL\sin\theta + L\cdot F(t)$$

$$\ddot{\theta} + \underbrace{\frac{c}{M}}_{2\beta}\dot{\theta} + \underbrace{\frac{g}{L}}_{\omega_0^2}\sin\theta = \underbrace{\frac{F_0}{ML}}_{\gamma\omega_0^2}\cos\omega t$$

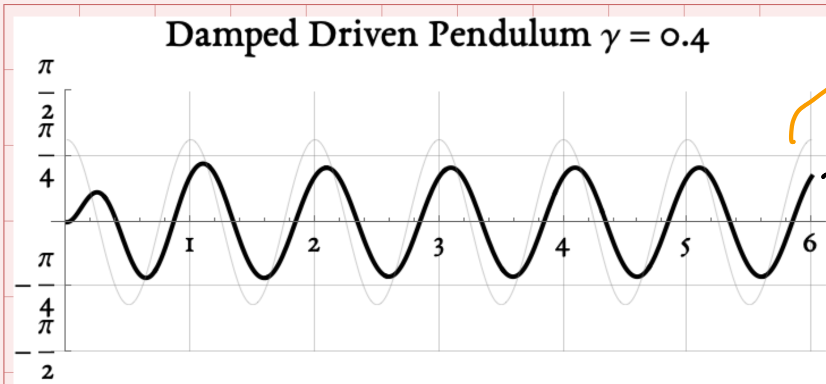
$$\boxed{\ddot{\theta} + 2\beta\dot{\theta} + \omega_0^2\sin\theta = \gamma\omega_0^2\cos\omega t}$$

β : damping parameter [1/sec]

ω_0 : natural frequency [1/sec]

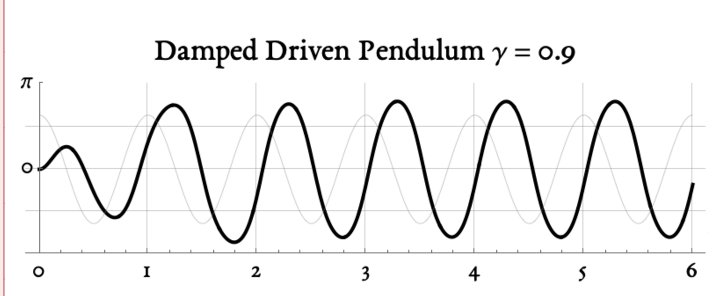
γ : drive strength [–] = F_0/mg

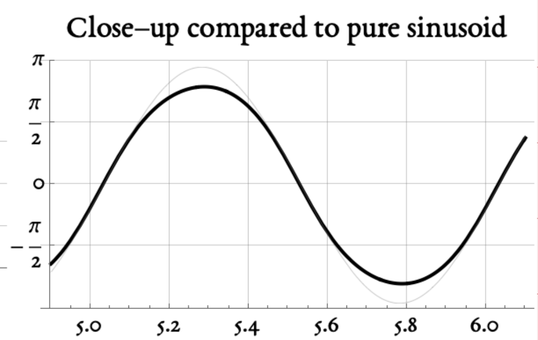
γ : tuning parameter

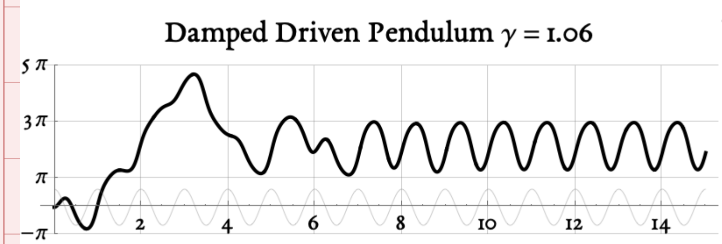


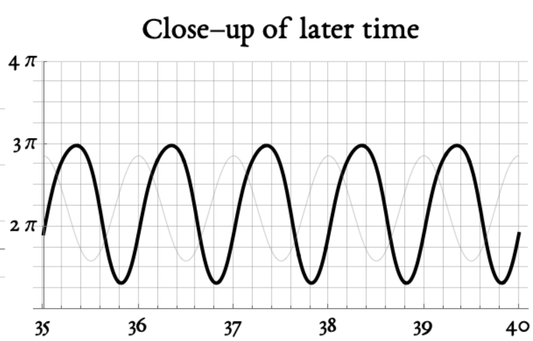
driving force [re-scaled]

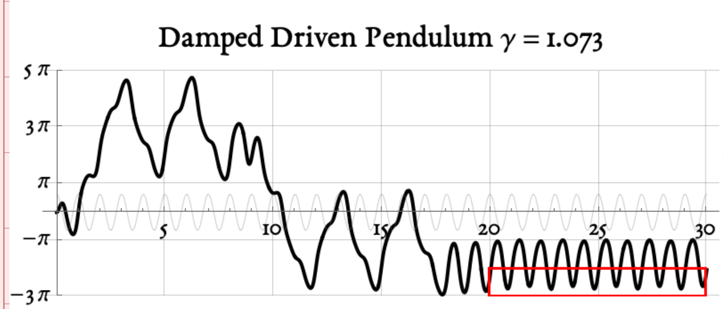
angle of pendulum

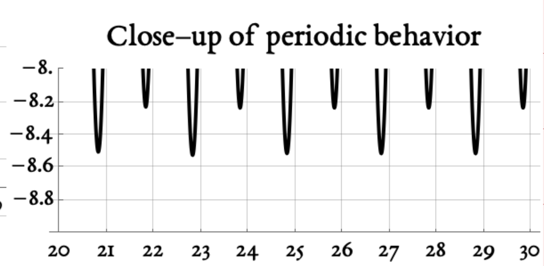


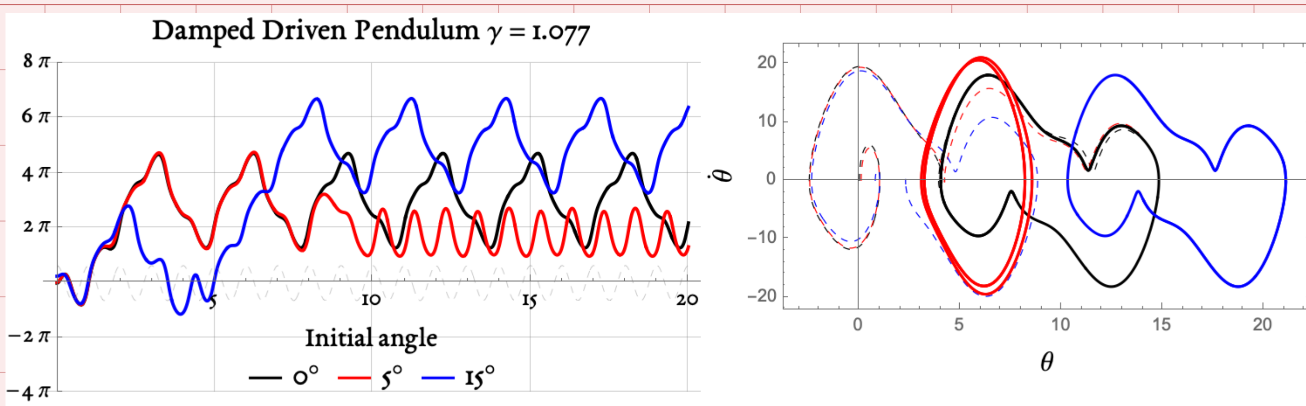




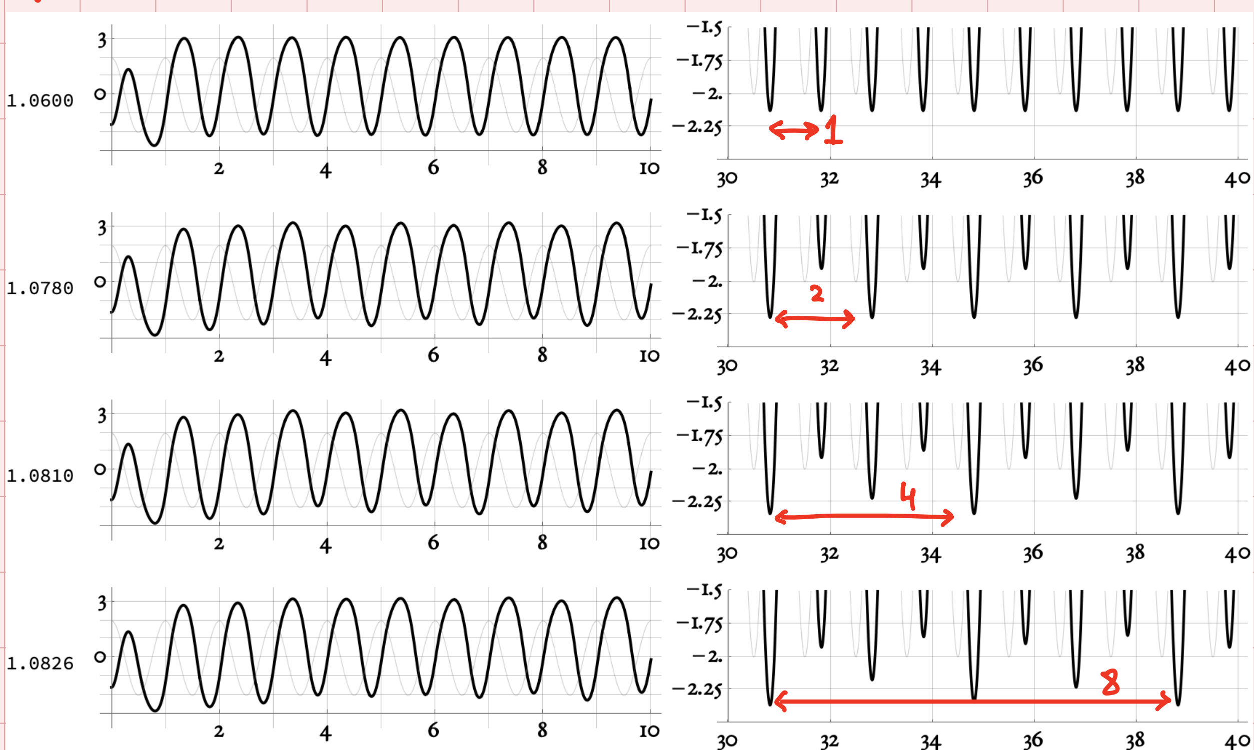




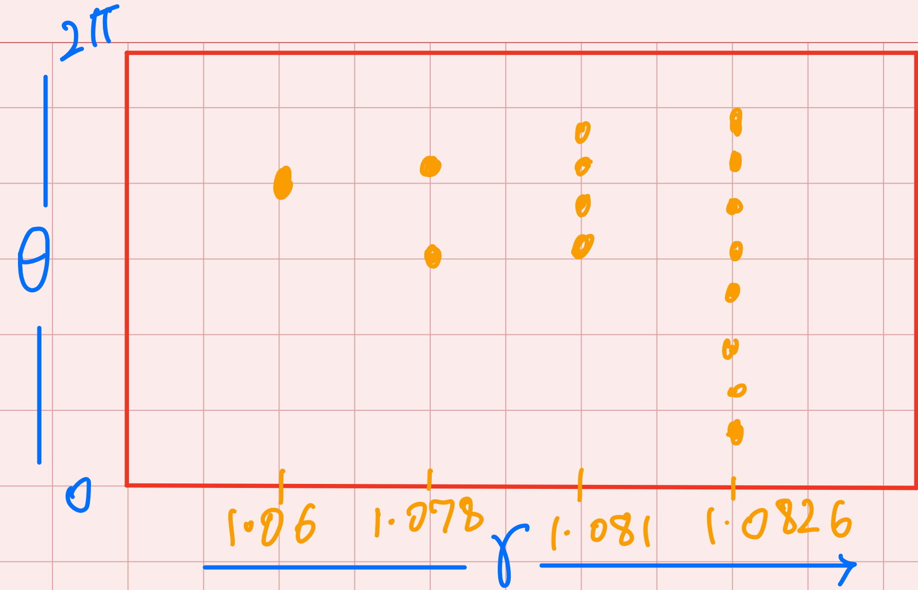




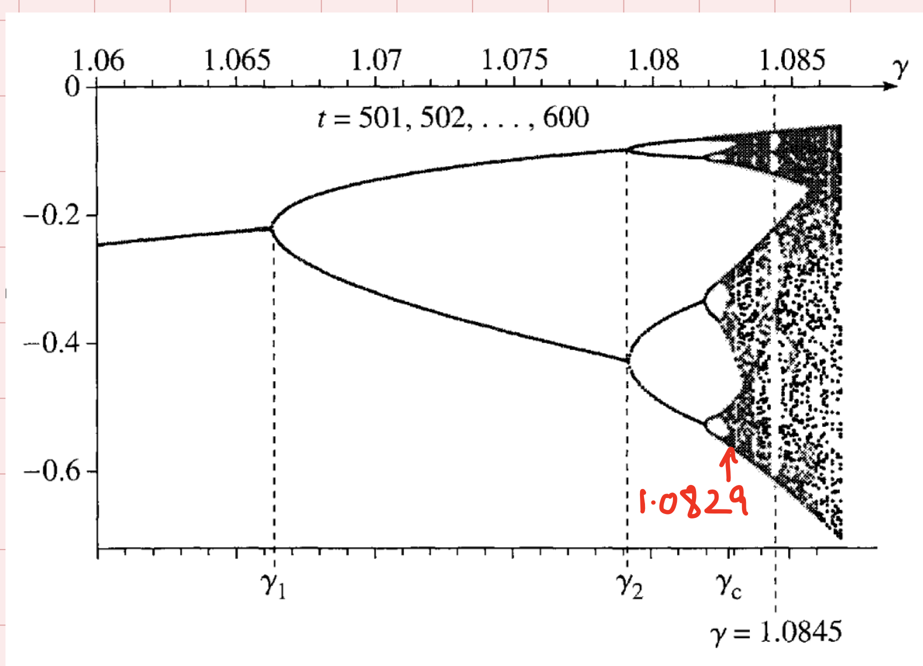
γ θ vs. time



Orbit diagram: For each γ, start with same initial condition. Integrate to find $\theta(t)$ to long times. Check the value of θ every 1 time unit. Organize results into an "orbit diagram". e.g. Plot the last 1000 time units.



$\gamma = 1.06$



Period-doubling route to chaos.

$\gamma_1 \longrightarrow \gamma_2 \longrightarrow \gamma_3 \longrightarrow \cdots$

$$\lim_{n\to\infty} \frac{\gamma_{n+1} - \gamma_n}{\gamma_n - \gamma_{n-1}} = \frac{1}{\delta}, \quad \boxed{\delta = 4.669}$$