Wed, Apr 30 Lecture 26

→ Nonlinear differential equations describe most of the physical world. Linear equations are usually approximations e.g. spring close to rest length; small deflections of a beam; constant 'g' for gravity.

→ The number of dimensions of phase space needed to describe real systems' dynamics can get very large. (degrees of freedom) Explicit time dependence adds another dimension to phase space. Most differential equations in physics are 2nd or 4th order.

→ Nonlinearity + "high"-dimensionality ⟶ possibility of chaos. How nonlinear? e.g. A pendulum to large angles $\theta \to \sin\theta$
How high-dimensional? $n > 2$ is enough.

→ Partial differential equations are like ∞-dimensional ordinary differential equations.

→ In the 19th century, it was thought that physics, in principle, had been solved. Initial conditions and differential eqns in ⟶ future behavior out.
"Laplace's Demon"

→ Quantum physics and general relativity complicated this picture at v. small and v. large scales. But until the discoveries of Lorenz in the 1960s, people thought that classical physics, on which engineering is based, did not suffer from such 'problems'.

→ The discovery of chaos — strictly a mathematical phenomenon — in numerous physically important equations has broken the spell of Laplace's Demon. It turns out that an adequate knowledge of the initial conditions is not enough to guarantee adequate knowledge of future behavior. — even if you know the governing equations and their solutions are guaranteed to exist and be unique.

→ The degree of unpredictability is so great that chaotic systems — despite being deterministic, not random — show certain features of random processes. e.g. long-time behavior of logistic map resembles long random walks.

→ We therefore learn that there is often a pretty stark limit on how long we can predict physical phenomena for, even if the physics is well-understood. Small uncertainties in measurement will propagate exponentially. Better measurement tools only delay this inevitable divergence of initially nearby trajectories very slightly.

→ So, the question of whether we live in a deterministic world is complicated by the presence of chaos in those equations that we know to be good models of physical phenomena. (Sometimes, even in the simplest eqs.) Even if the world is deterministic — i.e. even if the future state of the world is a function of current state of the world — it often appears to behave randomly if chaos is present in the governing equations.