A Fast Explicit Method for Simulating Flexible Fish-Like Swimmers in Inviscid Flow

Emad Masroor [1] Hodjat Pendar [2]

[1]Department of Engineering, Swarthmore College [2]Department of Mechanical Engineering, Virginia Tech

Inviscid models of flexible fins

Thin flexible bodies at moderate Reynolds numbers and low angle of attack can be modeled as a bound vortex sheet representing the attached boundary layer and a free vortex sheet representing the wake. The body can be modeled as a nonlinear elastica with governing equation [1]

$$R_1 \partial_{tt}\zeta = \partial_s T\hat{s} - R_2 \partial_s\left(\partial_s \kappa \hat{n}\right) - [p]\hat{n}, \tag{1a}$$

and the bound vortex sheet's evolving strength is found by an unsteady form of Bernoulli's equation,

$$\partial_t \gamma + \partial_s((\mu - \tau)\gamma) = \partial_s[p]. \tag{1b}$$

The system is closed with the enforcement, at n 'check points', of a constraint equation which states that the normal component of the local fluid velocity must be identical to the normal component of the local body velocity, i.e.

$$\boldsymbol{v}_f \cdot \boldsymbol{e}_n = \boldsymbol{v}_b \cdot \boldsymbol{e}_n \tag{1c}$$

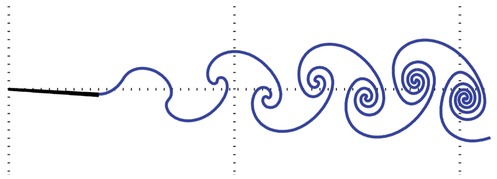


Figure 1. A thin flexible body with a vortex sheet wake from Alben (2009) J. Comp. Phys.

The no-penetration condition (1c), which requires the fluid velocity at the fin surface to be tangent to the body, constitutes a non-holonomic constraint on the motion of the fluid particles. This necessitates the iterative solution of a nonlinear system of equations at each time-step.

Panel method

We use a panel method coincident with an N-link chain connected by rotational springs of stiffness B_k, where the k^{th} link in the solid body dynamics is also the k^{th} panel of the bound vortex sheet.

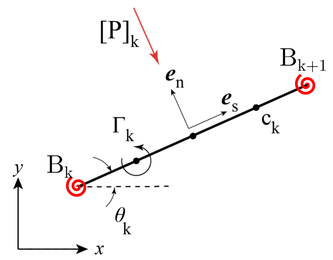


Figure 2. The k^{th} panel. The constraint equation (3) is satisfied at the check point located at the $3/4$-chord point, while the time-varying point vortex is located at the $1/4$-chord point.

System of Equations

1. Newton's second law applied to the N-link chain:
$$M\ddot{\boldsymbol{\theta}} + A_p[\boldsymbol{P}] = N \tag{2a}$$
2. Discetized bound vortex evolution equation
$$[\boldsymbol{P}] = H\dot{\boldsymbol{\Gamma}} + d \tag{2b}$$
3. Acceleration constraint from (3)
$$C_\theta \ddot{\boldsymbol{\theta}} + C_\Gamma \dot{\boldsymbol{\Gamma}} = N_c \tag{2c}$$

- $\boldsymbol{\theta}$: Each link's angle to the horizontal
- $[\boldsymbol{P}]$: Pressure difference across the bound vortex sheet
- $\boldsymbol{\Gamma}$: Circulation of each panel

After eliminating the pressure, we get a system of $2N$ equations for $2N$ unknowns $\ddot{\boldsymbol{\theta}}$ and $\dot{\boldsymbol{\Gamma}}$ can be solved explicitly

The system of differential equations (2) is marched forward in time explicitly for a small time interval $\Delta t = 0.1$ using `ode45` with Baumgarte's stabilization technique [2]. At each interval, a new fixed-strength free vortex Γ^+ is placed near the trailing edge. The circulation of the new point vortex is determined by satisfying Kelvin's Circulation Theorem. The point vortices placed in the wake are advected using the Biot-Savart kernels of all vortices, free and bound. The size of the dynamical system is equal to $N + N_v$, where N_v is the number of free vortices in the wake and N is the number of panels into which the fin is discretized. Since the newly shed vortices have fixed strength, the evolution equations only require the inversion of an augmented mass matrix of size $2N$ at each time step.

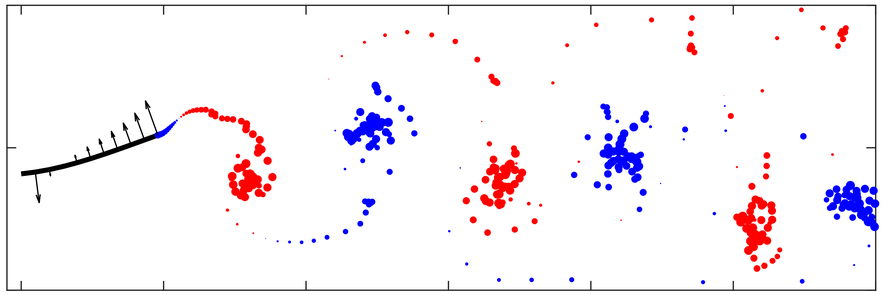
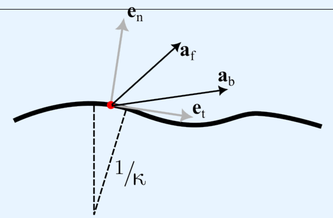


Figure 3. Visualization of the fin and point vortices in the wake. Arrows on the fin are proportional to the local pressure drop across it. Vortices with positive circulation are shown in blue, and negative circulation in red; the size of each dot is proportional to the magnitude of the circulation. Fin was impulsively started from rest with a sinusoidal pitch amplitude $A_{\theta_0} = 10°$ and normalized heave amplitude $A_{h_0} = 0.1$. Pitch lags heave by $90°$. Period of oscillation is 2π, and the solution is shown after 6.7 cycles; There are 421 free vortices in the wake. $N = 10$ panels were used.

Acceleration constraint

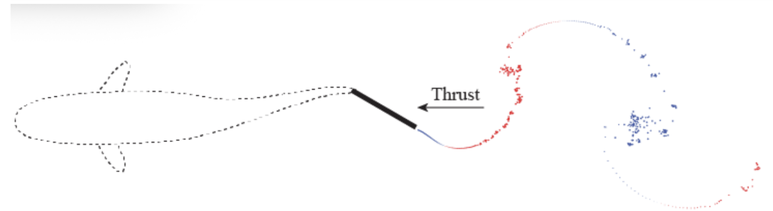
$$\boldsymbol{a}_f \cdot \boldsymbol{e}_n = \boldsymbol{a}_b \cdot \boldsymbol{e}_n + 2\dot{\theta} v_{\text{rel}} + \kappa v_{\text{rel}}^2 \tag{3}$$

- $\boldsymbol{a}_b$: body acceleration
- $\boldsymbol{a}_f$: fluid acceleration
- κ: local curvature
- v_{rel}: relative (tangential) velocity between the fluid and body



We consider a fluid particle sliding along the body. By equation (1c), the fluid velocity vector is constrained to be entirely in the tangential direction, and the difference between the fluid's normal velocity and the body's normal velocity is zero. The difference between the normal component of the fluid's acceleration and the normal component of the body's acceleration is given by the last two terms of equation (3), which gives us N equations involving the unknowns $\ddot{\boldsymbol{\theta}}$, $\dot{\boldsymbol{\Gamma}}$.

Modeling Caudal Fin-Driven Swimmers



Many aquatic swimmers produce thrust primarily from the actuation of their caudal fin [3], whereas the upstream body mostly contributes a drag force proportional to the square of the swimmer's speed. We model the caudal fin as a thin flexible filament that is pinned at its leading edge and subjected to combined heave and pitch. After a few cycles of impulsively-started motion, the fin achieves periodic thrust production corresponding to steady forward swimming.

Non-dimensional Parameters

R_1	0.5	Normalized Density of the fin
R_2	0.01	Normalized Rigidity of the fin
U_0	0.25	Normalized Free-stream speed
A_{θ_0}	$\pi/18$	Pitch amplitude
A_{h_0}	$1/5$	Normalized Heave amplitude
ϕ	$\pi/2$	Pitch lags heave by ϕ

Thrust and Power Coefficients

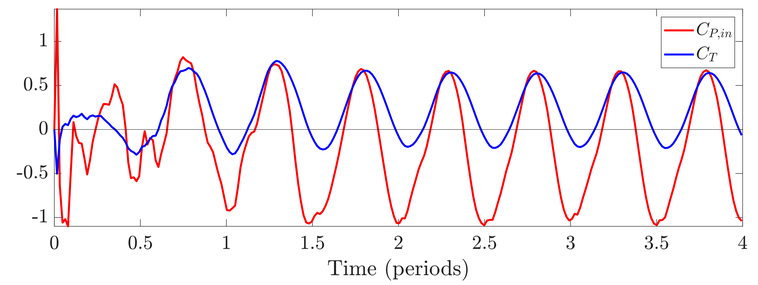


Figure 4. Input power ($C_{P,in}$) and thrust (C_T) coefficients for a simultaneously heaving and pitching fin.

References

[1] Silas Alben. "Simulating the dynamics of flexible bodies and vortex sheets". In: *Journal of Computational Physics* (2009).

[2] Farid Amirouche. "Numerical Stability of Constrained Multibody Systems". In: *Fundamentals of Multibody Dynamics: Theory and Applications*. 2006. (Visited on 05/11/2020).

[3] Richard Bainbridge. "Caudal Fin and Body Movement in the Propulsion of Some Fish". In: *Journal of Experimental Biology* (Mar. 1, 1963). (Visited on 07/08/2024).

emasroo1@swarthmore.edu