Higher-order pitchfork bifurcations In practice, systems with a subcritical pitchfork bifurcation don't actually go off to $\pm\infty$; instead, higher-order terms play a stabilizing role. Consider the system

$$\dot{x} = rx + x^3 - x^5, \tag{1}$$

where r is a single parameter that accounts for all the qualitatively different dynamics. Phase portraits are shown below for two values of r.

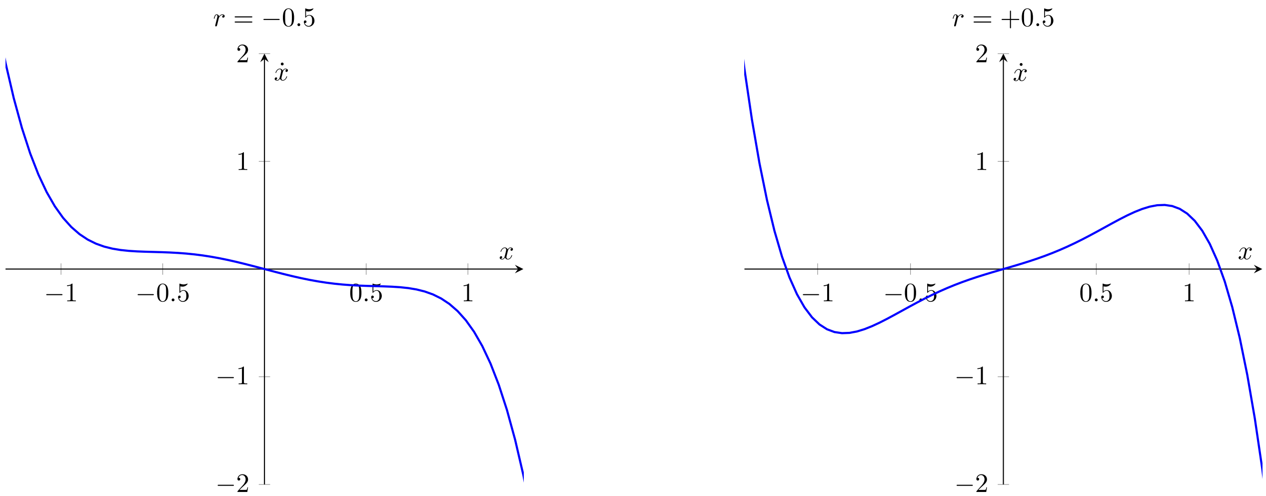


Figure 1: Phase portraits for $\dot{x} = rx + x^3 - x^5$.

✍ Draw arrows and fixed points to complete the phase portraits above.

✍ Draw some possible trajectories below. Indicate fixed points with horizontal lines.

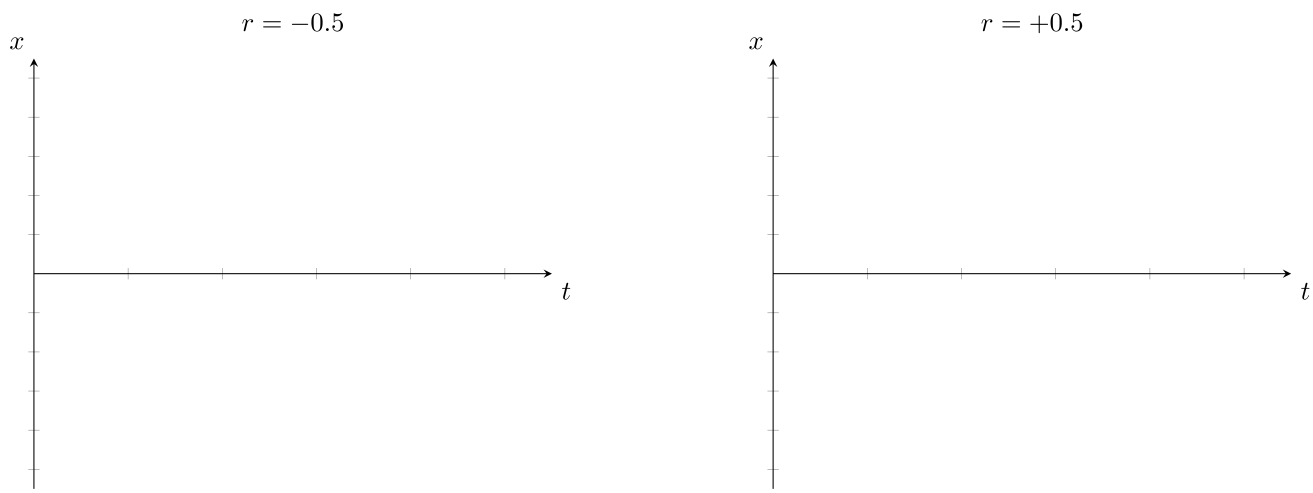
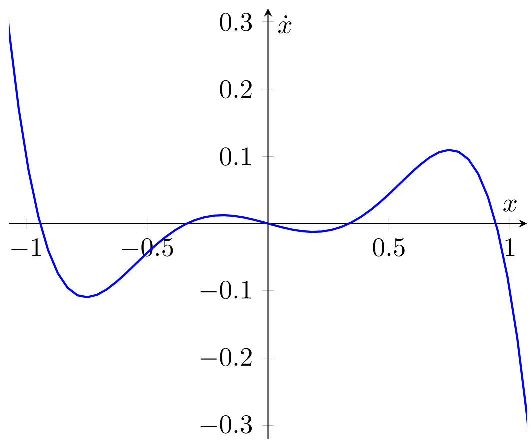


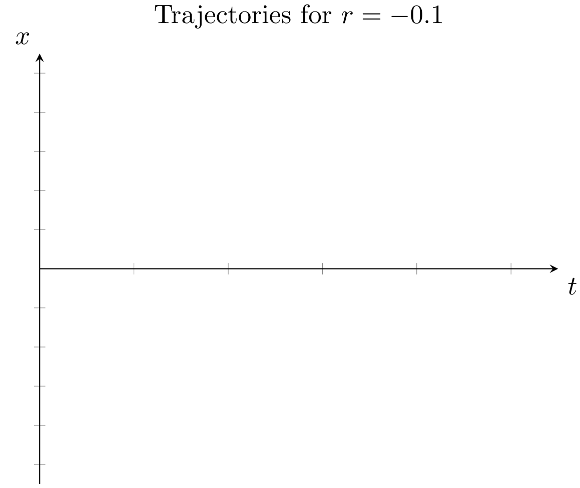
Figure 2: Trajectories for $\dot{x} = rx + x^3 - x^5$.

Next, we will consider the case when $r = -0.1$.

Phase portrait for $r = -0.1$



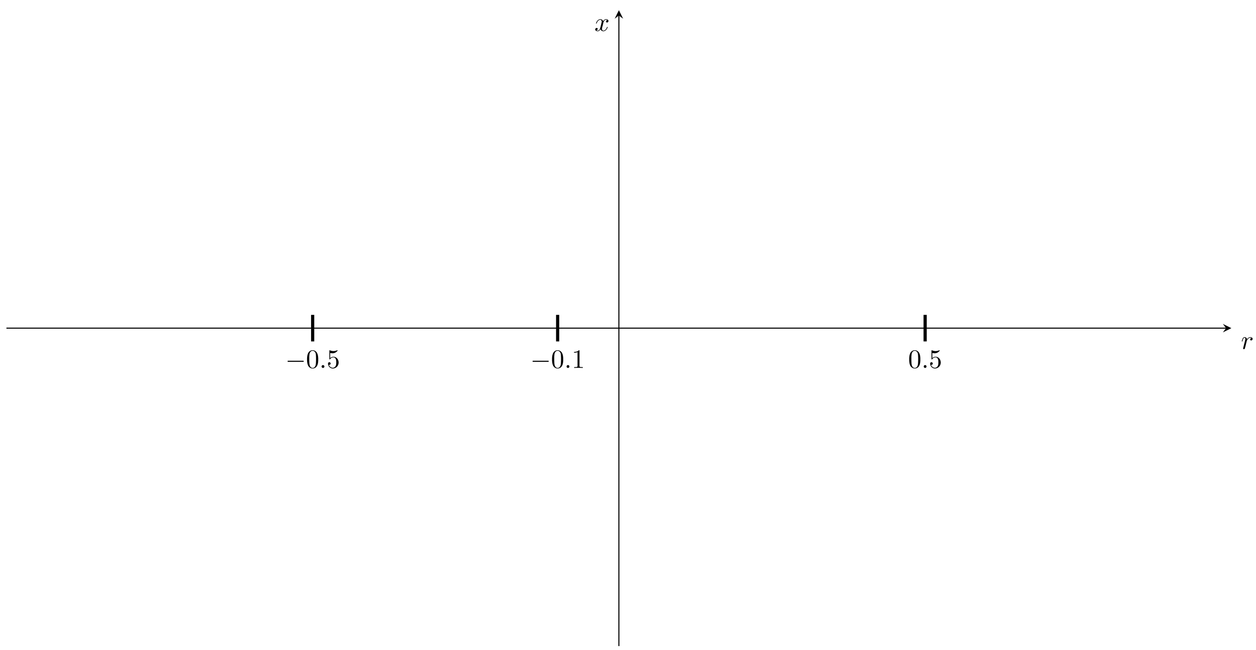
Trajectories for $r = -0.1$



✍ Complete the phase portrait and sketch some possible trajectories.

✍ Use what you have learned to fill in some points on the bifurcation curve shown below.

Bifurcation diagram for $\dot{x} = rx + x^3 - x^5$



✍ Use the Mathematica 'Manipulate' panel at `https://tinyurl.com/higherorderbifurcation1` to determine some other points on the bifurcation diagram.

✍ Do this for at least one value of $r < -0.5$, one value in the range $-0.5 < r < -0.1$, one in the range $-0.1 < r < 0.5$, and one value of $r > 0.5$.

✍ Note the stability and instability of each point you draw.

✍ Attempt to connect the points using a curve or curves.