Non-dimensionalization Consider the two equations

$$\dot{u} = au + bu^3 - cu^5, \tag{2a}$$
$$\dot{x} = rx + x^3 - x^5. \tag{2b}$$

These two equations are similar in their polynomial form, except that (2a) has three parameters whereas (2b) has only one. It is possible to show that (2b) is a 're-scaled' or nondimensionalized form of (2a). To see how this works, let us re-write the above equations explicitly,

$$\frac{\mathrm{d}u}{\mathrm{d}t} = au + bu^3 - cu^5, \tag{3a}$$
$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = rx + x^3 - x^5, \tag{3b}$$

where we have chosen a different time variable, τ, for x than for u. This is because both the dependent variable and the independent variable will be re-scaled.
Now, let

$$x \equiv \frac{u}{U}, \text{ and} \tag{4a}$$
$$\tau \equiv \frac{t}{T} \tag{4b}$$

for some constants U and T. You can think of U and T as some characteristic values of u and t respectively; it doesn't matter what they are, as long as they have the same units as the quantities that they are dividing. These two quantities, U and T, are *constants* and do not change with time; $\mathrm{d}U$ and $\mathrm{d}T$ will be zero.

✍ Substitute (4) into (3a) to arrive at an equation for $\frac{\mathrm{d}x}{\mathrm{d}\tau}$ in terms of a, b, c, T and U.

✍ Equate the coefficients of your equation from the last part to the coefficients of (3b), and use the resulting equations to solve for r, U and T in terms of a, b and c.

✍ Assuming that u has units of length ('L') and t has units of time ('T'), show that the units of a, b and c are T^{-1}, $L^{-2}T^{-1}$ and $L^{-4}T^{-1}$ respectively. Then, check your expressions for T and U from above to ensure that T and U have the correct units.