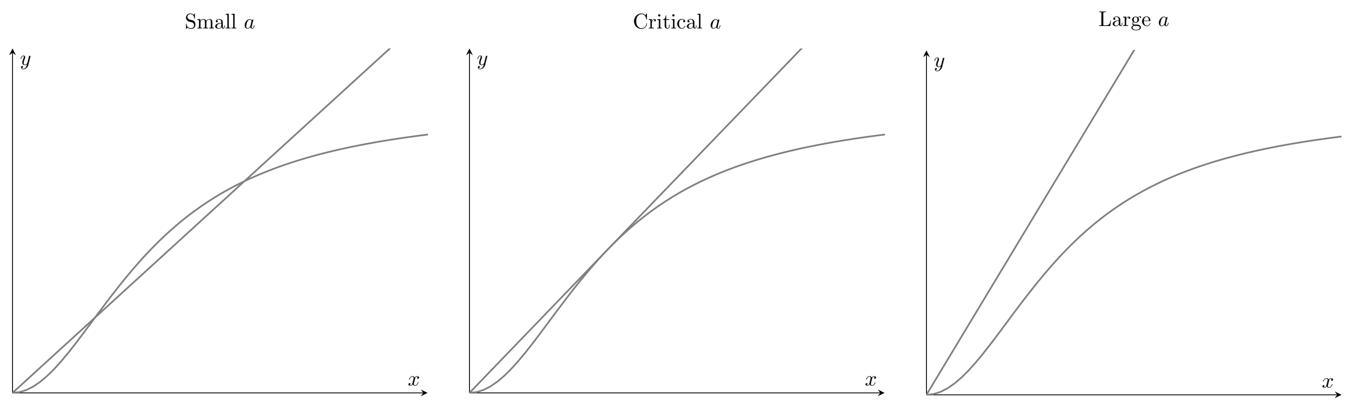
Consider the dynamical system

$$\dot{x} = -ax + y$$
$$\dot{y} = \frac{x^2}{1+x^2} - by$$

The figure below shows the nullclines (i.e., the curves where $\dot{x} = 0$ or $\dot{y} = 0$) for this system:

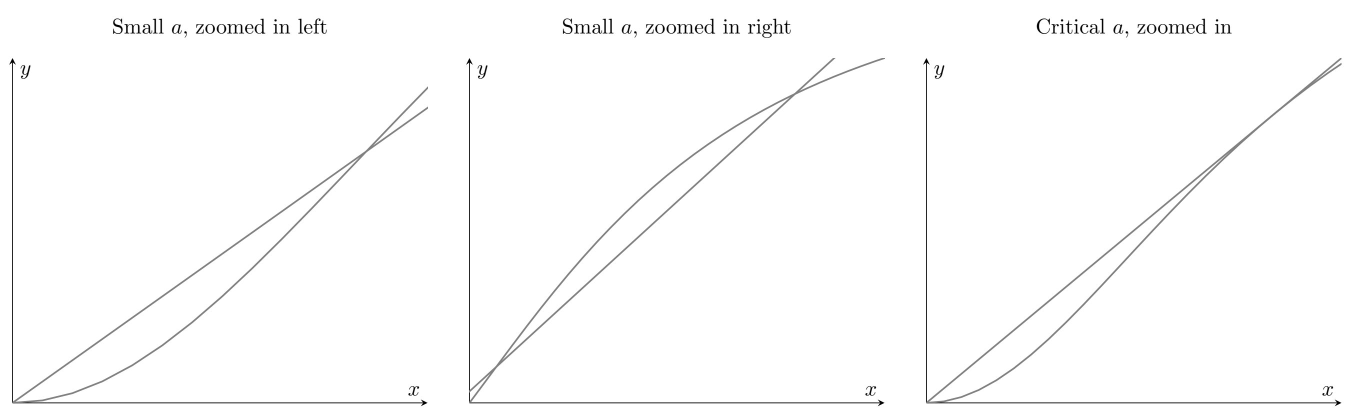
$$y = ax, \text{ and}$$
$$y = \frac{x^2}{b(1+x^2),}$$

for three different values of a while b is kept fixed at $b = 2$. You can see this interactively at `https://tinyurl.com/E91SaddleNode`.



✍ Sketch arrows to show the phase portrait in each case.

✍ You should use the 'zoomed-in' phase portraits below to clarify the phase portraits in the narrow space between the nullclines.



✍ Considering the idea of 'topological equivalence', sketch exaggerated versions of the phase portraits for the cases 'small a' and 'critical a', in which the nullclines are more widely separated. These phase portraits will be quantitatively **inaccurate**, but should be topologically equivalent to what you drew above.

Consider the system

$$\dot{x} = \mu x + y + \sin x$$
$$\dot{y} = x - y$$

✍ Write down an equation that must be satisfied by the coordinates of the fixed points of this system.

✍ Write down, in symbolic form, the Jacobian for this system, in terms of μ.

✍ Classify the fixed point $(0,0)$ based on the value taken by μ. What type is it, and for what value of μ does the answer change?

✍ For small distances away from $(0, 0)$, some other fixed points may also exist. Determine an expression for the coordinates of these other fixed points in terms of μ. You may need to use the series expansion of $\sin x$ or $\cos x$, as appropriate.

✍ Sketch the phase portrait 'before', 'during', and 'after' the bifurcation that occurs in this system. You may use the help of software such as `pplane`.