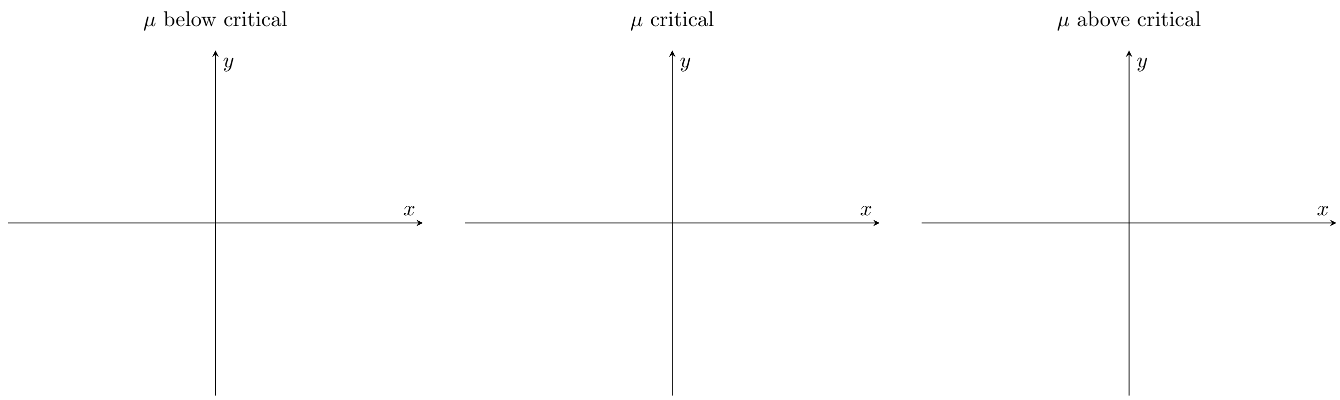
Consider the dynamical system

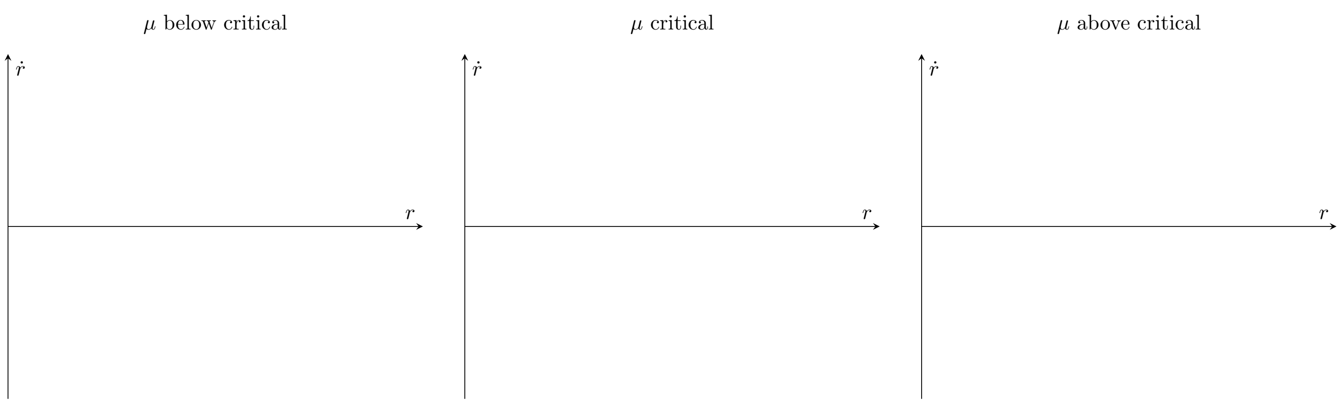
$$\dot{r} = \mu r - r^3 \tag{1a}$$
$$\dot{\theta} = \omega + br^2 \tag{1b}$$

An interactive view of the phase portrait for this system is shown at `https://tinyurl.com/E91supercriticalhopf`. This interactive version performs the necessary coordinate transformation from r, θ to x, y coordinates.

✍ Identify a bifurcation, and sketch the phase portrait before, during, and after the bifurcation.



✍ Also plot $\dot{r}$ as a function of r for each of the three cases above.



✍ What is the radius of the limit cycle that is formed after the bifurcation?

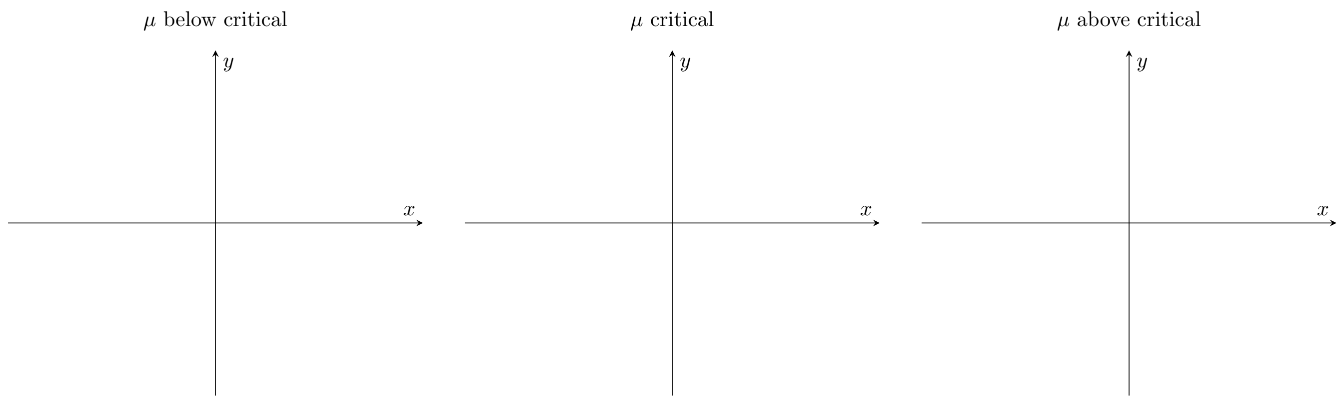
✍ Is the limit cycle stable or unstable? Re-write (1) so that the Hopf bifurcation leads to a limit cycle with the opposite stability.

Consider the dynamical system

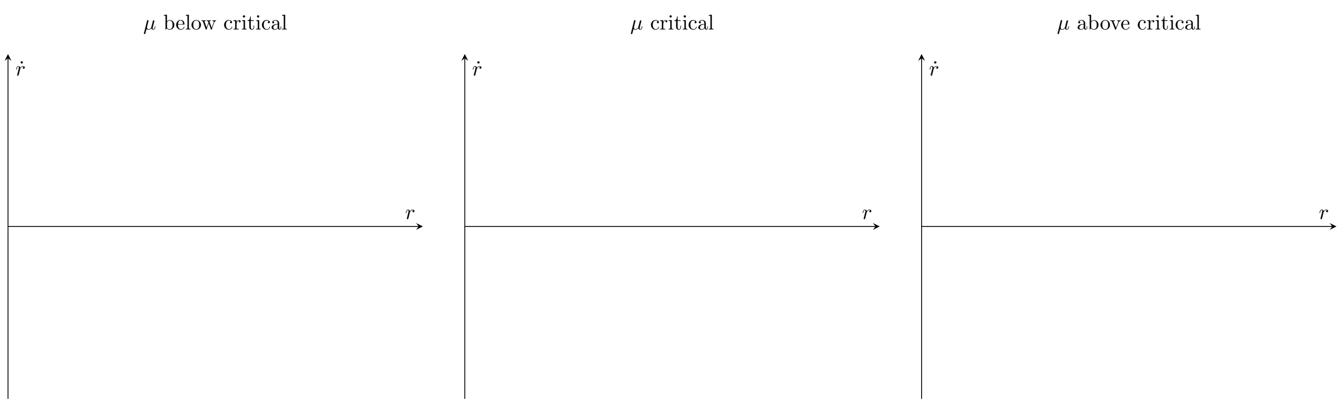
$$\dot{r} = \mu r + r^3 \tag{2a}$$
$$\dot{\theta} = \omega + b r^2 \tag{2b}$$

An interactive view of the phase portrait for this system is shown at `https://tinyurl.com/E91subcriticalhopf`. This interactive version performs the necessary coordinate transformation from r, θ to x, y coordinates.

✍ Identify a bifurcation, and sketch the phase portrait before, during, and after the bifurcation.



✍ Also plot $\dot{r}$ as a function of r for each of the three cases above.



✍ What is the radius of the limit cycle that is formed after the bifurcation?

✍ Is the limit cycle stable or unstable?

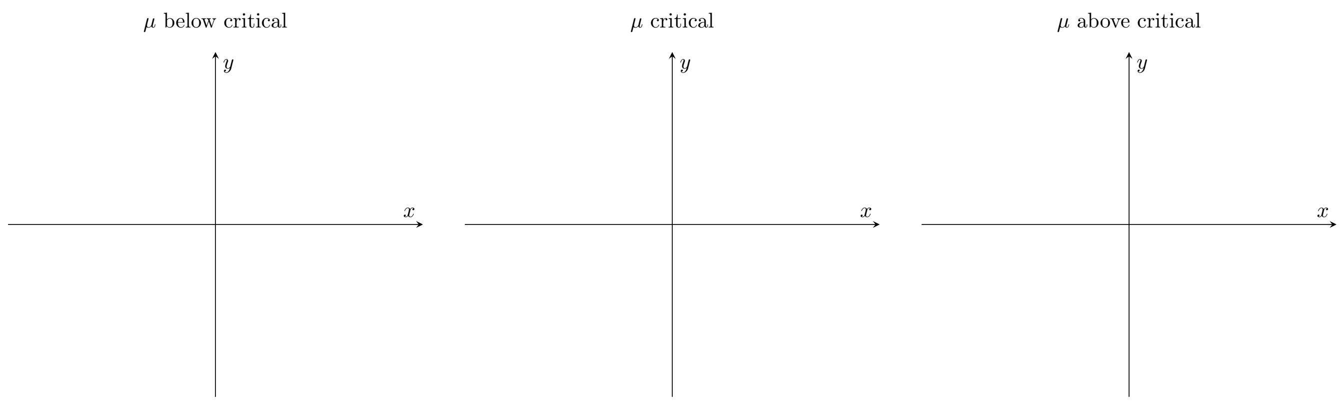
Consider the dynamical system

$$\dot{r} = \mu r + r^3 - r^5 \tag{3a}$$

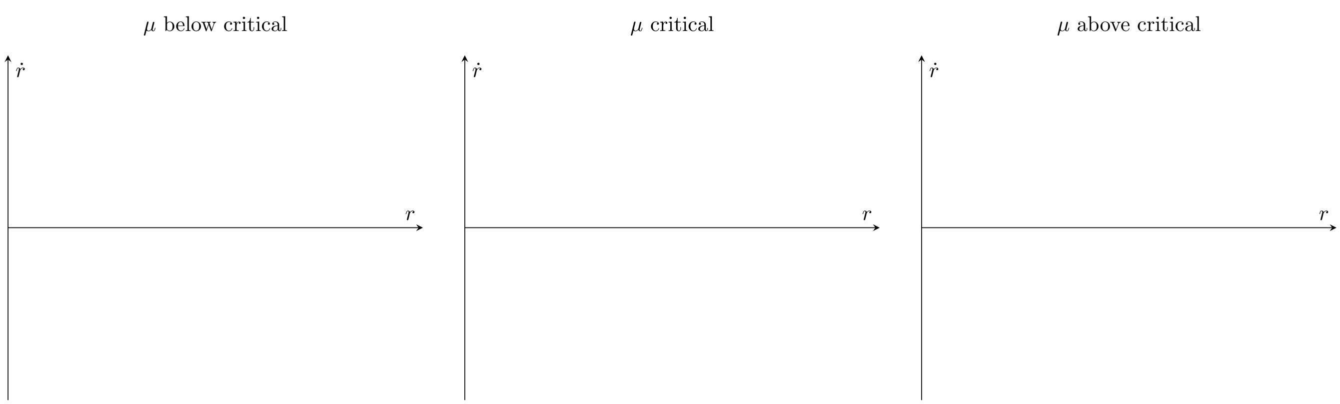
$$\dot{\theta} = \omega + b r^2 \tag{3b}$$

An interactive view of the phase portrait for this system is shown at `https://tinyurl.com/E91subcriticalhigherorderhopf`. This interactive version performs the necessary coordinate transformation from r, θ to x, y coordinates.

✍ Identify a bifurcation near $\mu = -0.25$, and sketch the phase portrait before, during, and after the bifurcation. Note that there is more than one bifurcation in this system; we are only concerned with the one that occurs near $\mu = -0.25$.



✍ Also plot $\dot{r}$ as a function of r for each of the three cases above.



✍ What is the radius of the limit cycles that are formed after the bifurcation?

✍ Are the limit cycles stable or unstable?

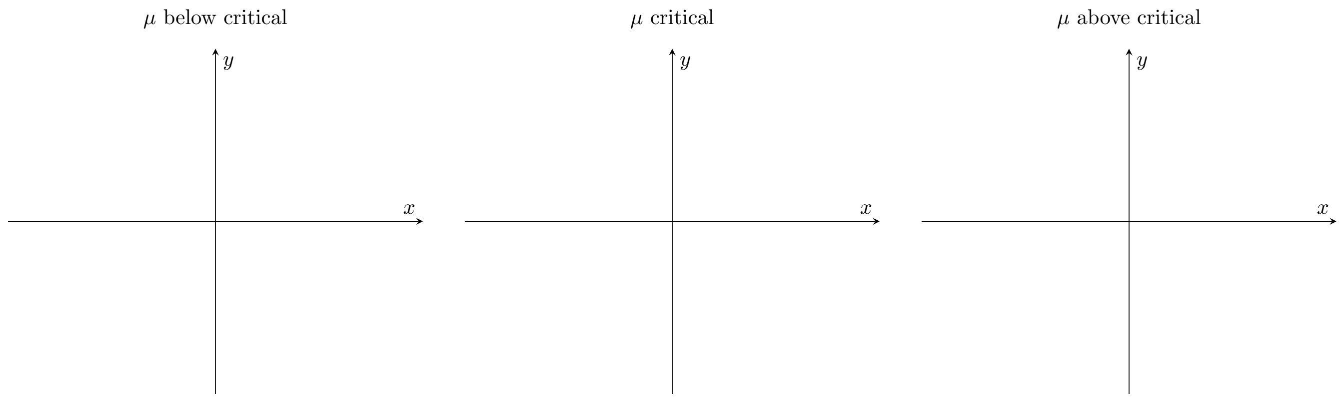
Consider the dynamical system

$$\dot{r} = r(1 - r^2) \tag{4a}$$

$$\dot{\theta} = \mu - \sin\theta \tag{4b}$$

An interactive view of the phase portrait for this system is shown at `https://tinyurl.com/E91infiniteperiodhopf`. This interactive version performs the necessary coordinate transformation from r, θ to x, y coordinates.

- ✍ Identify a bifurcation and sketch the phase portrait before, during, and after the bifurcation. It is recommneded that you use values of μ that are relatively close to the critical value.



- ✍ Label the fixed points and limit cycles, and visually determine the stability of each.

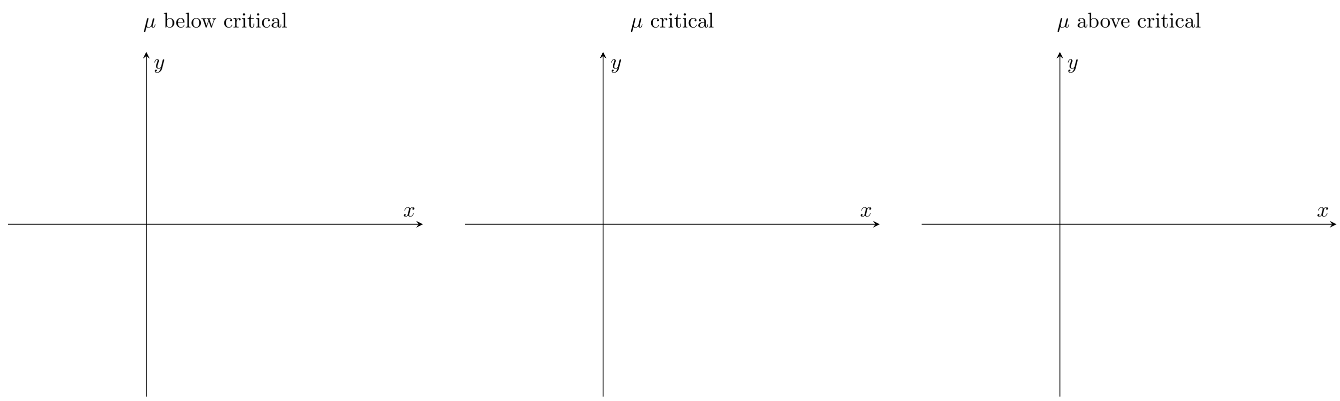
Consider the dynamical system

$$\dot{x} = y \tag{5a}$$

$$\dot{y} = \mu y + x - x^2 + xy \tag{5b}$$

An interactive view of the phase portrait for this system is shown at `https://tinyurl.com/E91homoclinicbifurcation`. You are also encouraged to plot this system on `pplane`.

- ✍ Identify a bifurcation near $\mu \approx -0.8645$, and sketch the phase portrait before, during, and after the bifurcation. It is recommneded that you use values of μ that are relatively close to the critical value.



- ✍ Label the fixed points and limit cycles, and classify them visually.