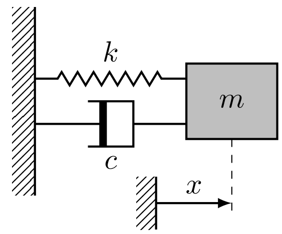
1. Governing equations for a simple harmonic oscillator. The simple harmonic oscillator is governed by the following second-order ordinary differential equation.

$$m\ddot{x} + c\dot{x} + kx = 0, \tag{1a}$$

where the symbols can be interpreted to have the meaning shown in the diagram below.



We can supplement the differential equation (1a) with two initial conditions, on the position and velocity respectively,

$$x(0) = x_0, \tag{1b}$$
$$\dot{x}(0) = v_0. \tag{1c}$$

to obtain a complete initial value problem (1).

(a) Identify the 'state variables' of the dynamical system given by (1a).
There are two state variables. The 'state variables' are the variables whose values define the 'state' of the system at any time t; they can be thought of as the things whose rates of change are given by the differential equation(s) under consideration.
If you have trouble thinking through this part, move on to the next one, and come back to this later.

(b) Write down a first-order dynamical system with $n = 2$ that is equivalent to eq. (1a). Do this in three different ways.
   i. Write two separate scalar differential equations.
   ii. Write a vector differential equation, i.e., write an equation where the left hand side is a column vector, and the right hand side is also a column vector.
   iii. Write a matrix-vector differential equation, i.e., write an equation where the left hand side is a column vector, and the right hand side is a matrix (that does **not** contain the state variables) multiplied by a column vector (whose elements are the state variables).

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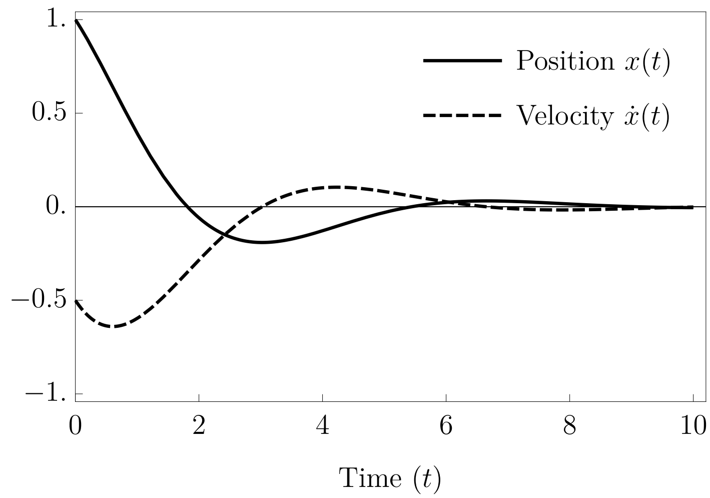
2. Analytical solution of the governing equations for a simple harmonic oscillator. In this problem, you will use your knowledge of differential equations to solve the differential equation (1a). For this problem, you should ignore the initial conditions (1b) and (1c). Instead, use the ansatz

$$x(t) = \exp \lambda t, \tag{2}$$

where λ has both a real and an imaginary part. All physical quantities are to be interpreted as the real part of corresponding complex numbers. Thus, both $x(t)$ and $\dot{x}(t)$ will in general be complex, and we interpret them physically using their real part.
Analytically solve the equation (1a) for the position $x(t)$ and velocity $\dot{x}(t)$ using the ansatz (2). Give your answer in terms of k, m, c and t (not λ). There may be more than one correct expression for the position and the velocity.

3. Plotting the analytical solution. For the case when $m = k = c = 1$, show that one of the solutions to (1a) together with (2) can be graphically represented as shown here. To answer this question successfully, you should reproduce the plot below using your answer to problem 2. Note that this problem does **not** ask you to find a **numerical solution** to (1a). Instead, you are asked to use a computer to plot the analytical solution to (1a) that was found in problem 2.



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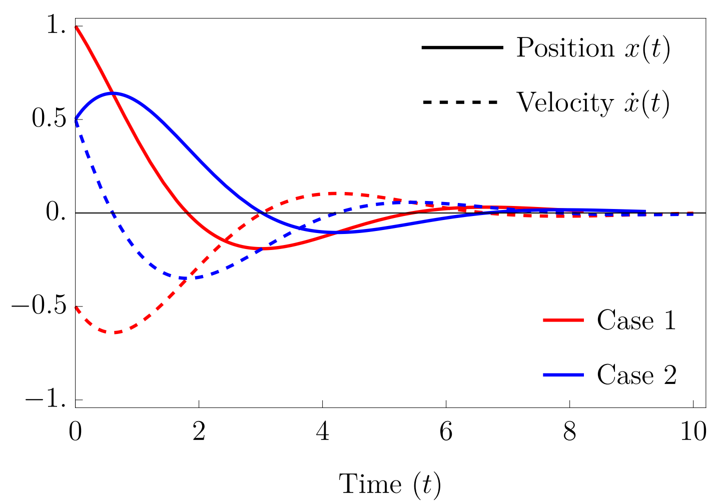
4. Numerical solution of ODEs (see tutorial at `https://emadmasroor.github.io/classes/E91_S25/Resources/NumericalSolution`). For the case when $m = k = c = 1$, numerically solve the differential equation (1a) using a program of your choice, and plot the resulting solutions — both $x(t)$ and $\dot{x}(t)$ — for $t \in [0, 10]$. Do this for two cases:

   - Case 1: $x(0) = 1, \quad \dot{x}(0) = -1/2$, and
   - Case 2: $x(0) = \dot{x}(0) = 1/2$.



You should be able to reproduce the figure above. Turn in the code you use to solve the problem as an additional file.

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5. Self-assessment. Complete the survey located at this link.