1. A geometric way of thinking about dynamics. For each of the following dynamical systems of dimension $n = 1$,

   (a) Draw a phase portrait similar to the one drawn in class for $\dot{x} = \sin x$.

   (b) Identify the fixed points, and decide whether each is an 'attractor' or a 'repeller'.

   (c) List the qualitatively different things that can happen to x over time depending on where on $\mathbb{R}$ the flow is initialized (i.e., depending on the initial condition).

   Answer (a) through (c) for:

   $$\dot{x} = x - x^3 \tag{1}$$
   $$\dot{x} = e^x - \cos x \tag{2}$$

   Your answers should be in the form of a neatly drawn sketch or plot, and complete sentences. For example, you could write

   > "System (2) has two fixed points: an attractor at A and a repeller at B as shown in the diagram. If x is initialized between 0 and 3, it will increase until it reaches B."

2. For the logistic equation

   $$\dot{N} = rN\left(1 - \frac{N}{K}\right), \tag{3}$$

   (a) determine the value of N at which N increases the fastest, in terms of parameters r and K.

   (b) Numerically solve (3) for a reasonable value of r and of K with three different initial conditions of your choosing. A computer-generated plot is sufficient to answer this part. Turn in your code also.

3. The growth of certain tumors can be modeled using the equation

   $$\dot{N} = -aN\log(bN), \tag{4}$$

   where log stands for the natural logarithm. $N(t)$ is proportional to the number of cells in the tumor.

   (a) What could the parameters a and b represent in real-world terms?

   (b) For $a = 1.2$ and $b = 1.0$, draw a phase portrait and sketch the graph of $N(t)$ for three initial values: $N(0) = \{0.1, 0.6, 1.6\}$. You should produce these plots with a computer program of your choice.

   (c) Explain (in complete sentences) what happens to the number of tumor cells as time passes for each of the initial values from 3b.

4. Consider the dynamical system

   $$\dot{x} = (x - 1)(x - 2). \tag{5}$$

   Which of the following, if any, are possible trajectories $x(t)$ arising from this dynamical system? For those that are possible, indicate on a phase portrait of system (5) where they are initialized. For those that are not possible, explain how you arrived at this conclusion.