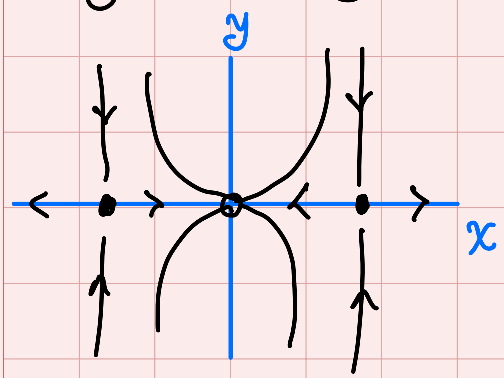
$$\dot{x} = -x + x^3$$
$$\dot{y} = -2y$$

1) Find fixed pts.
2) Characterize each.



What kind of fixed pt. is it?

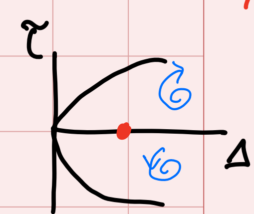
calculate Jacobian matrix, evaluate at each fixed pt.

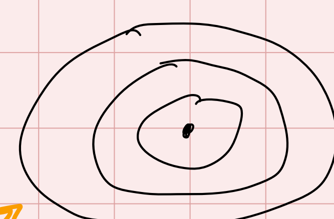
where is this matrix on τ-Δ plane?

Mon, Feb 24　Lecture 10

Ex.
$$\dot{x} = -y + ax(x^2+y^2)$$
$$\dot{y} = x + ay(x^2+y^2)$$
→ (0,0) is a fixed pt. Classify its stability

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{bmatrix} \text{ evaluate at } (0,0) \longrightarrow \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

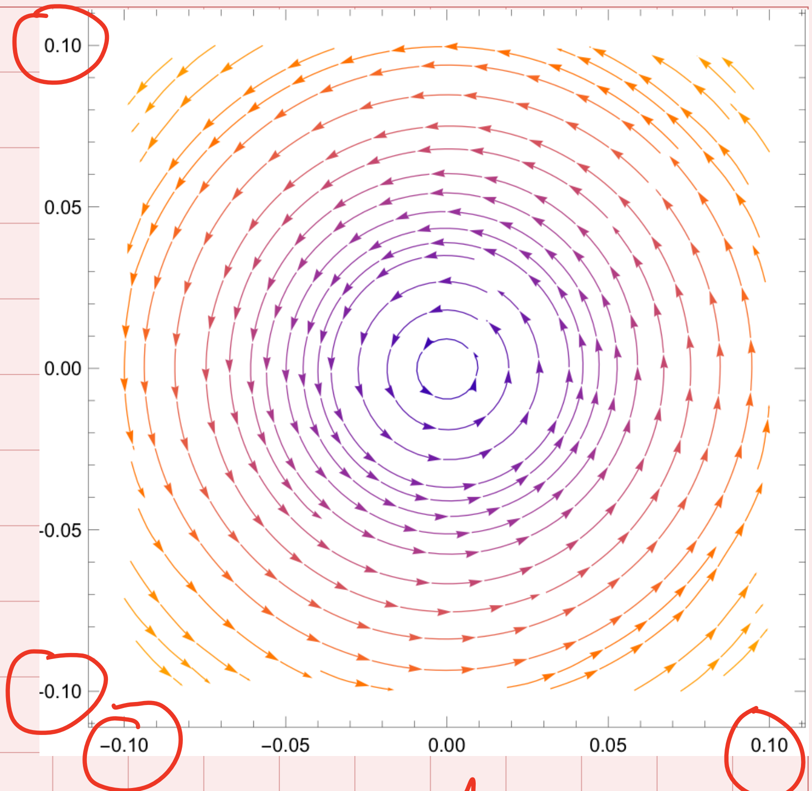




Linear theory predicts

$$\dot{x} = -y + ax(x^2+y^2)$$
$$\dot{y} = x + ay(x^2+y^2)$$

$a = 1$

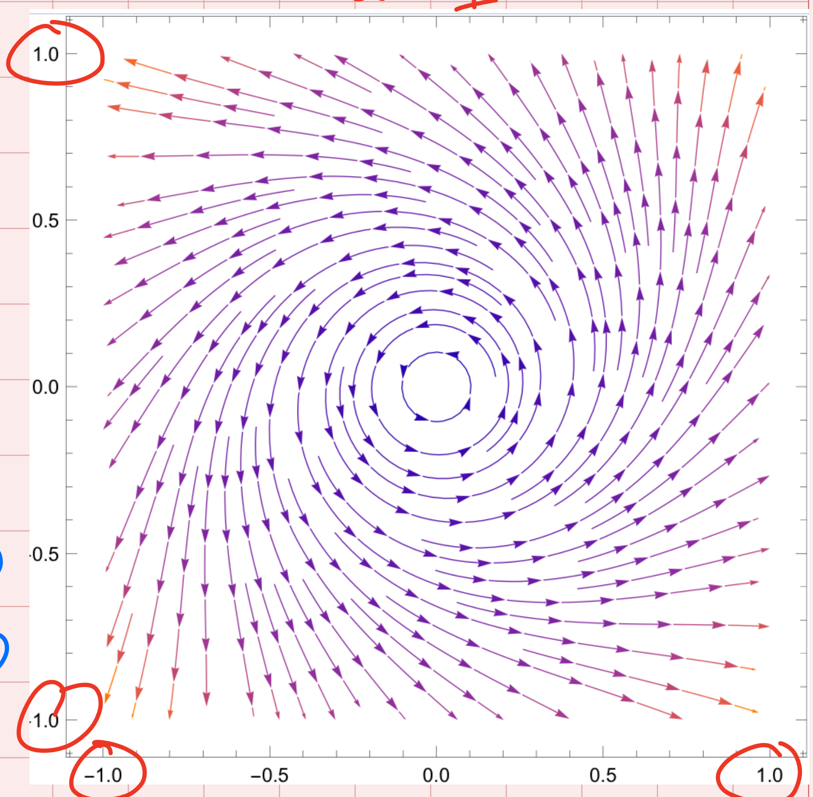


Near the origin, looks like a center.

But — notice the axes

... zoom out :

$a = 1$



and it looks like an unstable spiral.

Strogatz 6.3.2 proves that this system is like an

unstable spiral if $a > 0$

stable spiral if $a < 0$

(using r, θ coordinates)

Hyperbolic Fixed Pts.

Fixed pts that remain unchanged, qualitatively, by small nonlinear terms, relative to their linearized phase portraits.

"Local phase portrait near a hyperbolic fixed pt. is topologically equivalent to the phase portrait of its linearized version"

a homeomorphism exists between the two.

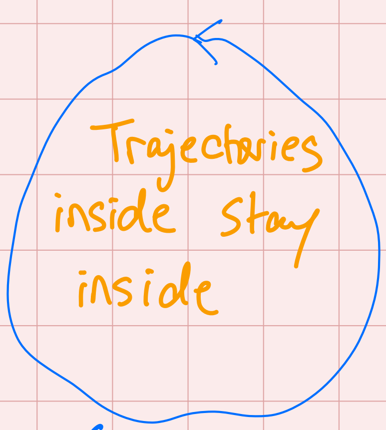
For hyperbolic fixed pts, all eigenvalues have non-zero real part.

↳ if one or more eigenvalue has zero real part, the fixed pt. is nonhyperbolic.

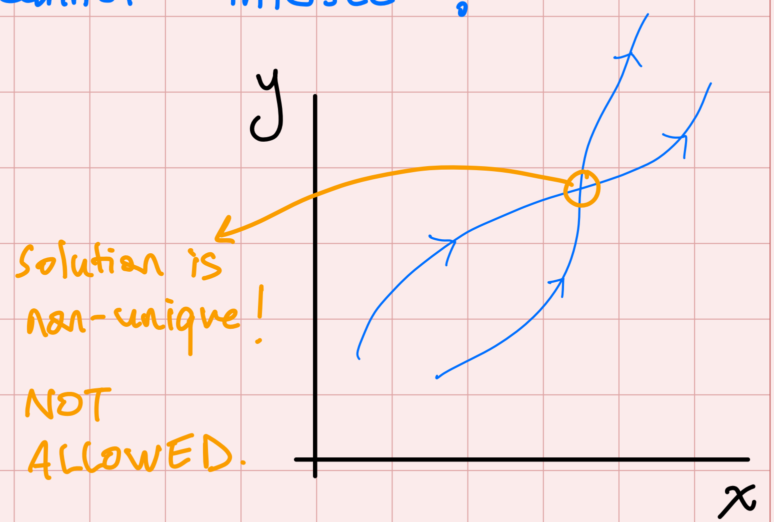
$$\dot{\underset{\sim}{x}} = f(\underset{\sim}{x}) \quad , \quad \underset{\sim}{x}(0) = \underset{\sim}{x}_0 \qquad \underset{\sim}{x} \in \mathbb{R}^n$$

if f is continuous and all partial derivatives of f are continuous on a subset $D \subset \mathbb{R}^n$, then for $\underset{\sim}{x}_0$ in D, the I.V.P above has a unique solution $\underset{\sim}{x}(t)$ at least for some time.

$\implies$ Trajectories cannot intersect!



closed orbit.



Lotka-Volterra Population Dynamics

- Two species competing for a resource (limited)
- Each species has a growth rate, carrying capacity

  → $\dot N = rN(1 - N/K)$ logistic eqn.

- Two logistic eqns + competition.

x: rabbits

y: sheep

rabbits grow faster than sheep

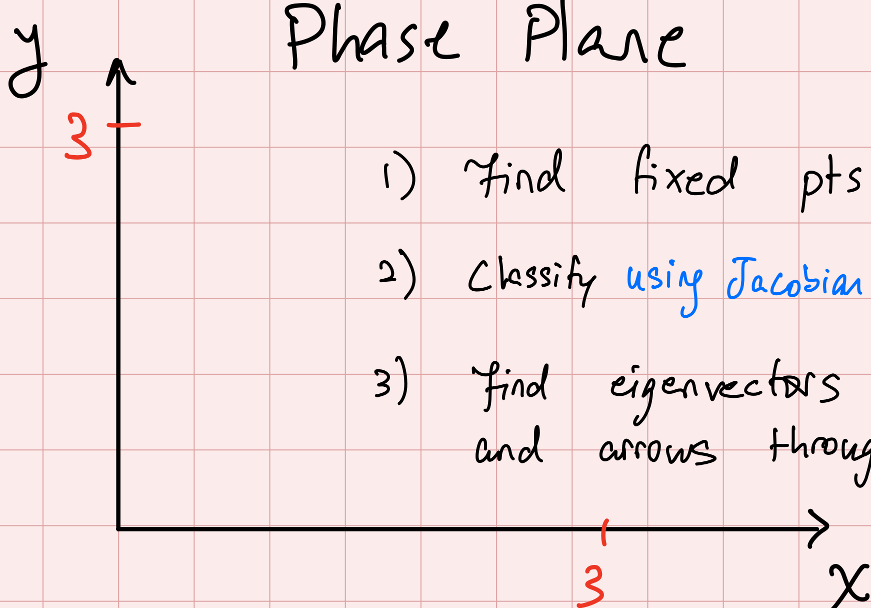
$$\dot x = x\left(3 - x/2 - 2y\right)$$

$$\dot y = y\left(2 - x - y\right)$$

Rabbits have higher carrying capacity

Sheep stronger than rabbits.

Phase Plane



1) Find fixed pts $(0,2)$ $(0,0)$ $(6,0)$ $(2/3, 4/3)$
2) Classify using Jacobian
3) Find eigenvectors to put lines and arrows through the fixed pts.