Wed, Feb 26 Lecture 11

Building up nonlinear phase plane using linearizations at each fixed pt.

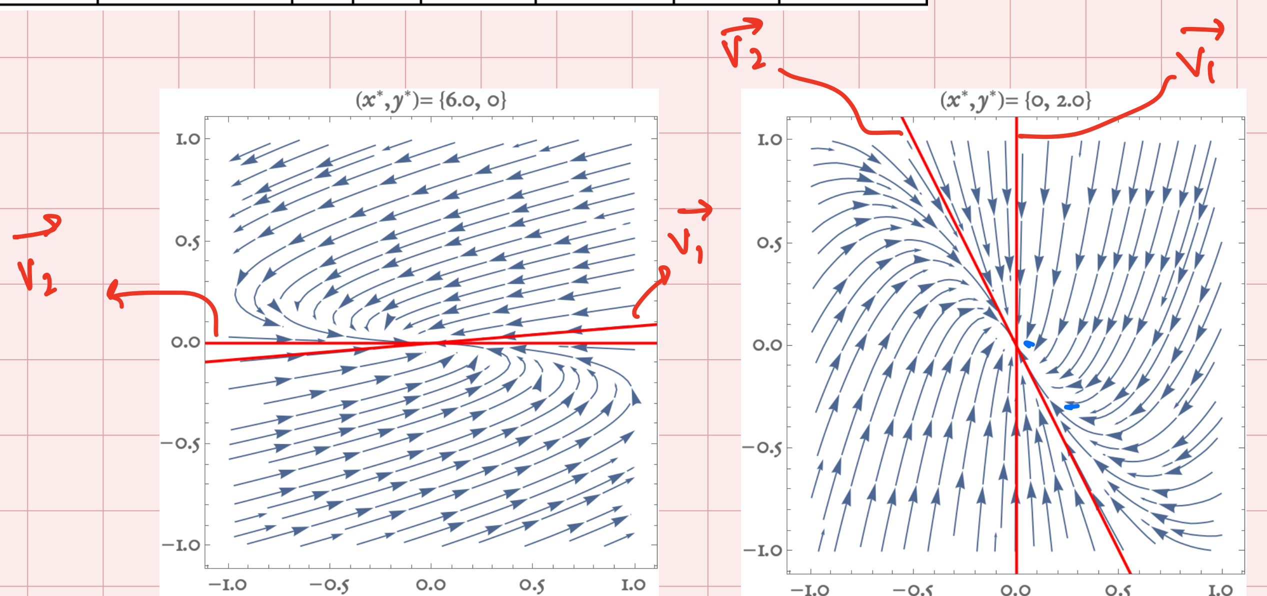
$(0,2) \longrightarrow \quad A = [\quad] \quad \lambda_{1,2} = \ldots \quad \vec{v}_{1,2} = [\ ]$

$(6,0) \longrightarrow$

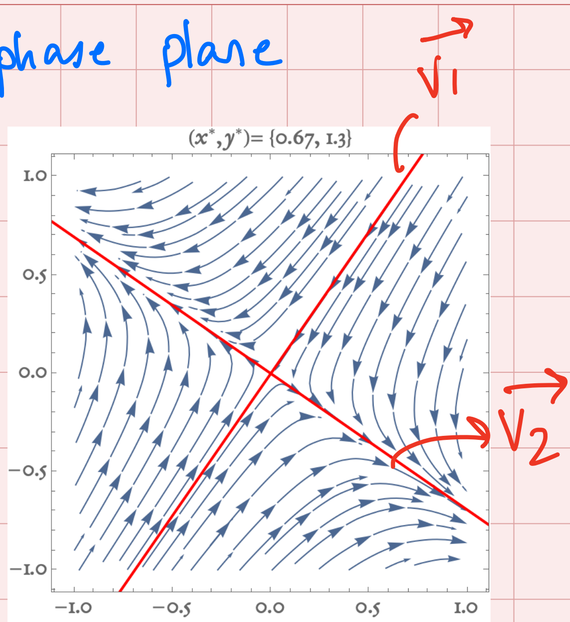
$(0,0) \longrightarrow$

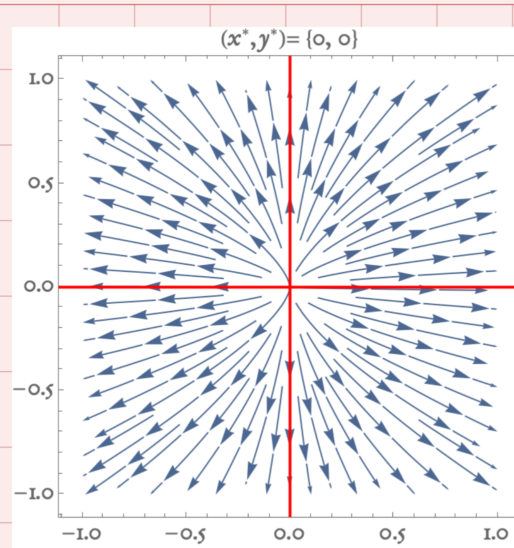
$(2/3, 4/3) \longrightarrow \quad \lambda = \{-2.26, +0.591\}$

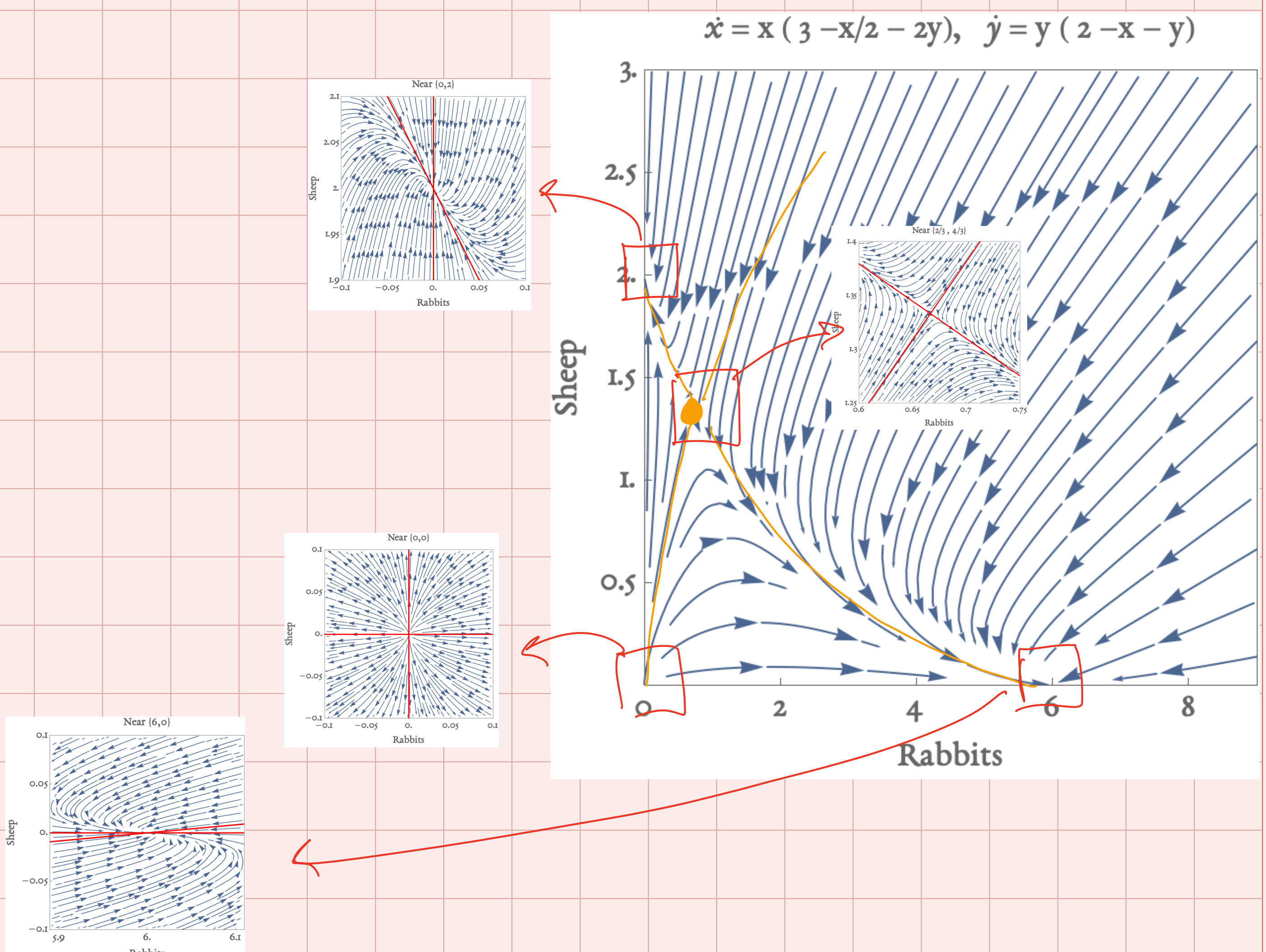
Fixed Point	Jacobian	τ	Δ	λ_1	$\vec{v}_1$	λ_2	$\vec{v}_2$	
$\begin{pmatrix} 0 \\ 2 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 \\ -2 & -2 \end{pmatrix}$	-3	2	-2.00	$\begin{pmatrix} 0 \\ 1.00 \end{pmatrix}$	-1.00	$\begin{pmatrix} -1.00 \\ 2.00 \end{pmatrix}$	stable node
$\begin{pmatrix} \frac{2}{3} \\ \frac{4}{3} \end{pmatrix}$	$\begin{pmatrix} -\frac{1}{3} & -\frac{4}{3} \\ -\frac{4}{3} & -\frac{4}{3} \end{pmatrix}$	$-\frac{5}{3}$	$-\frac{4}{3}$	-2.26	$\begin{pmatrix} 0.693 \\ 1.00 \end{pmatrix}$	0.591	$\begin{pmatrix} -1.44 \\ 1.00 \end{pmatrix}$	saddle
$\begin{pmatrix} 6 \\ 0 \end{pmatrix}$	$\begin{pmatrix} -3 & -12 \\ 0 & -4 \end{pmatrix}$	-7	12	-4.00	$\begin{pmatrix} 12.0 \\ 1.00 \end{pmatrix}$	-3.00	$\begin{pmatrix} 1.00 \\ 0 \end{pmatrix}$	stable node
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$	5	6	3.00	$\begin{pmatrix} 1.00 \\ 0 \end{pmatrix}$	2.00	$\begin{pmatrix} 0 \\ 1.00 \end{pmatrix}$	unstable node



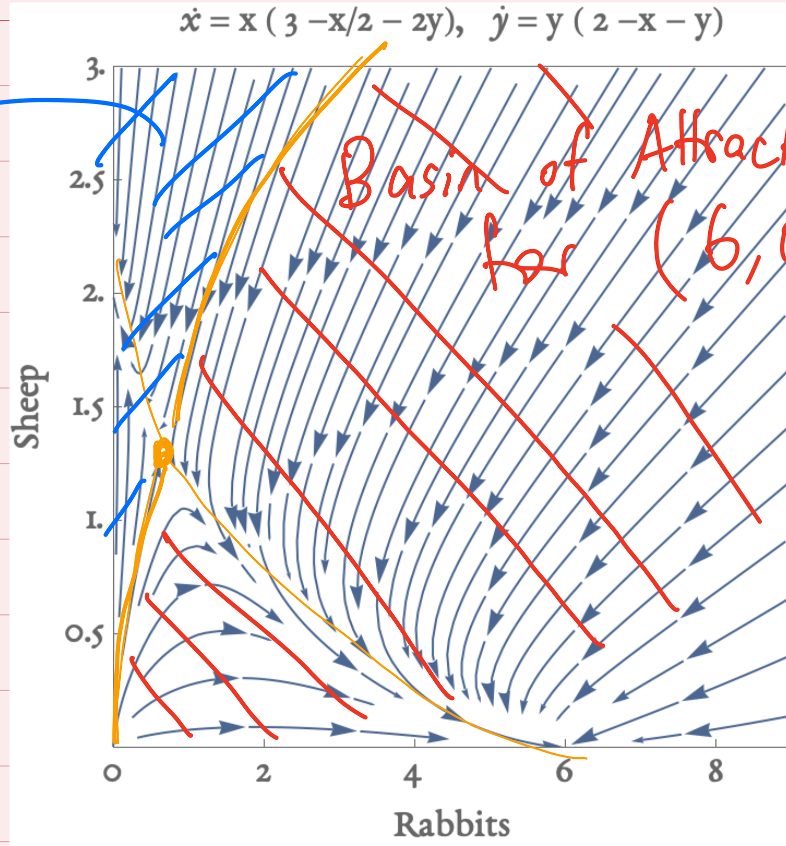
Linear phase plane







Basin of Attraction for $(0,2)$



Conservative Systems

Given $\dot{\underset{\sim}{x}} = f(\underset{\sim}{x})$, $\underset{\sim}{x} \in \mathbb{R}^n$, a conserved quantity is a real-valued continuous & scalar function $E(\underline{x}) \neq \text{const.}$ that is constant on trajectories, i.e. $\frac{dE}{dt} = 0$

If a system has a conserved quantity, it is called a conservative system.

$$\underbrace{\ddot{x}}_{\text{"accel."}} = \underbrace{x^3 - x}_{\text{"force" } F(x)}$$

Find a conserved quantity for this system.

$$F(x) = -\frac{dV}{dx}$$

potential $V(x)$

$$x^3 - x = -\frac{dV}{dx}$$

$$\int (x^3 - x)\,dx = \int - dV$$

$$\frac{x^4}{4} - \frac{x^2}{2} = -V + c$$

$$\Rightarrow \boxed{V(x) = \frac{x^2}{2} - \frac{x^4}{4} + c}$$

$$\ddot{x} = F(x) = -dV/dx$$

$$\ddot{x} + \frac{dV}{dx} = 0$$

$$\dot{x}\ddot{x} + \dot{x}\frac{dV}{dx} = 0$$

$$\frac{d}{dt}\Big(\underbrace{\tfrac{1}{2}\dot{x}^2 + V(x)}_{E(x,\dot{x})}\Big) = 0$$

we have found that $\frac{1}{2}\dot{x}^2 + V(x)$ is a conserved quantity.

① $m\ddot{x} + c\dot{x} + kx = 0$

None exists

② $\ddot{\theta} = \sin\theta$

find $V(x)$, not $V(x, \dot{x})$

$$m\ddot{x} + kx = 0$$

$$E = \frac{1}{2}\dot{\theta}^2 + \cos\theta$$

$$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$$