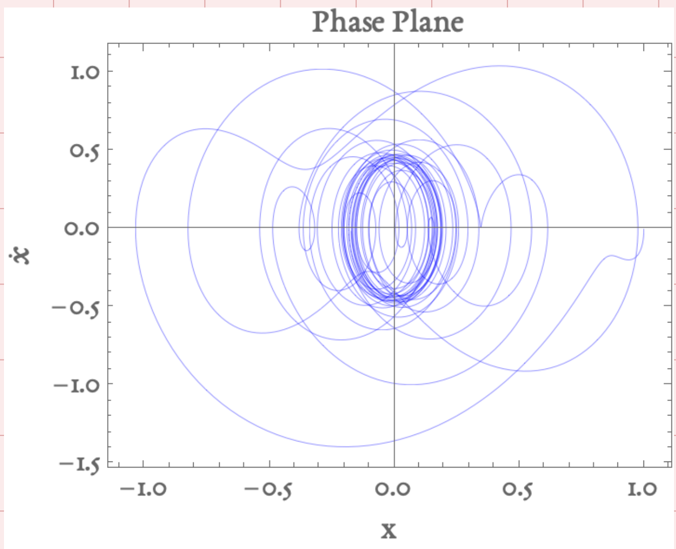
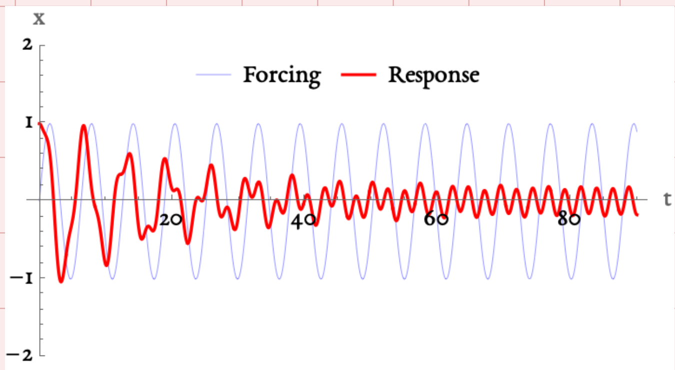
Wed, Mar 5 Lecture 13

Limit Cycles

$$m\ddot{x} + c\dot{x} + kx = \sin(\omega t)$$





This is not an $n=2$ dynamical system $\{\dot{\underset{\sim}{x}} = f(\underset{\sim}{x})\}$

An n^{th} order $\overbrace{\text{time-dependent}}^{=\text{nonautonomous}}$ eqn. is a special case of an $(n+1)^{\text{th}}$ order autonomous dynamical system

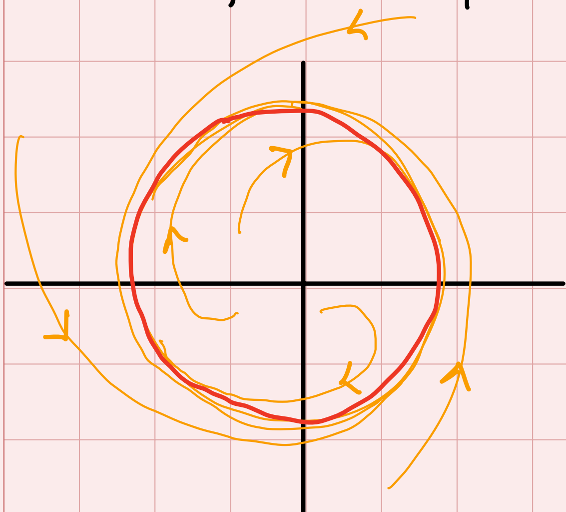
$$\ddot{x} + \dot{x} + x = \sin \omega t$$

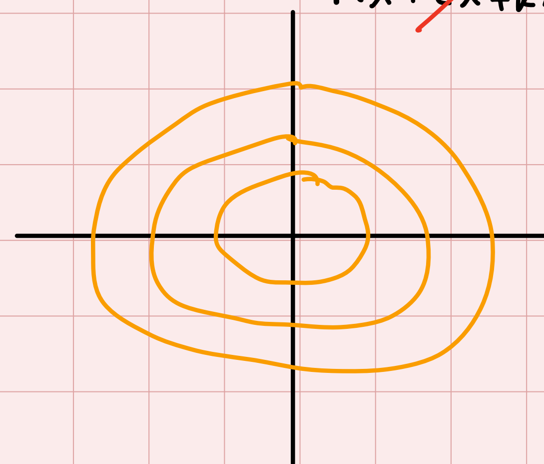
$$x_1 = x \qquad x_2 = \dot{x} \qquad x_3 = \omega t$$

$$\dot{\underset{\sim}{x}} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ -x_1 - x_2 + \sin x_3 \\ \omega \end{bmatrix}$$

Limit Cycle is an isolated closed (closed orbit) trajectory in phase space

$$m\ddot{x} + \cancel{c\dot{x}}^{\,0} + kx = 0$$

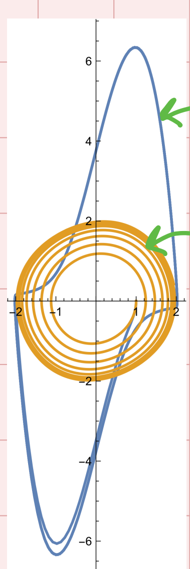




closed orbit that is a limit cycle.

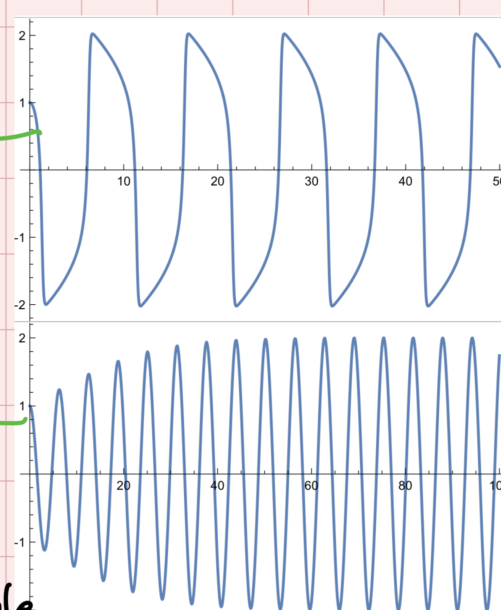
closed orbits but NOT limit cycles

<u>van der Pol oscillator</u> $\ddot{x} + \mu(x^2-1)\dot{x} + x = 0$



2 diff limit cycles

$\boxed{\text{A distinctive feature of nonlinear systems}}$



All trajectories approach the limit cycle (b/c it's <u>stable</u>). Opposite for unstable ones.