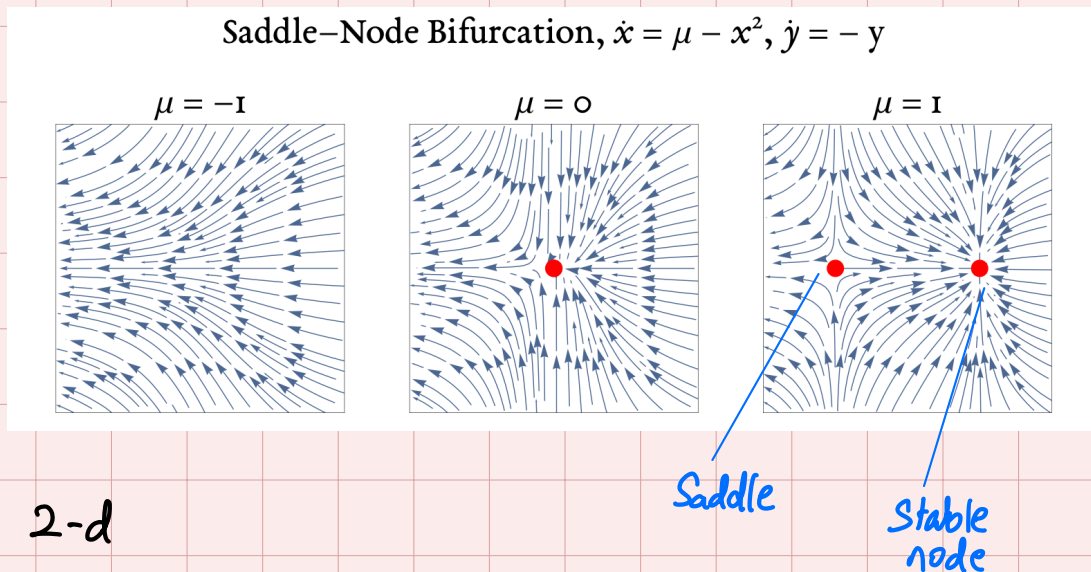


# Bifurcations

A bifurcation occurs when the topology of phase space changes (qualitatively)

$$\dot{x} = \mu - x^2$$

$$\dot{y} = -y$$



Saddle-node

Bifurcation in 2-d occurs in one dimension

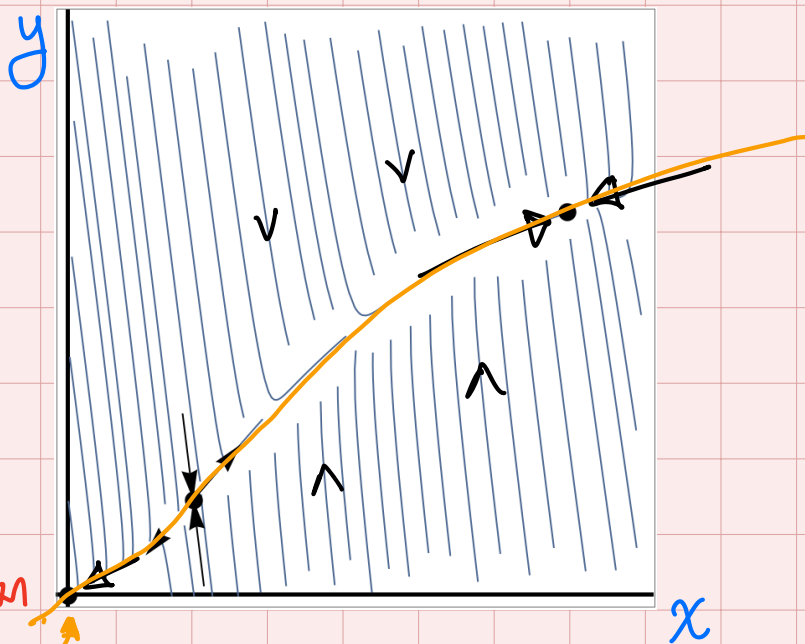
$$\dot{x} = -ax + y$$

$$\dot{y} = \frac{x^2}{1+x^2} - by$$

$$a, b > 0$$

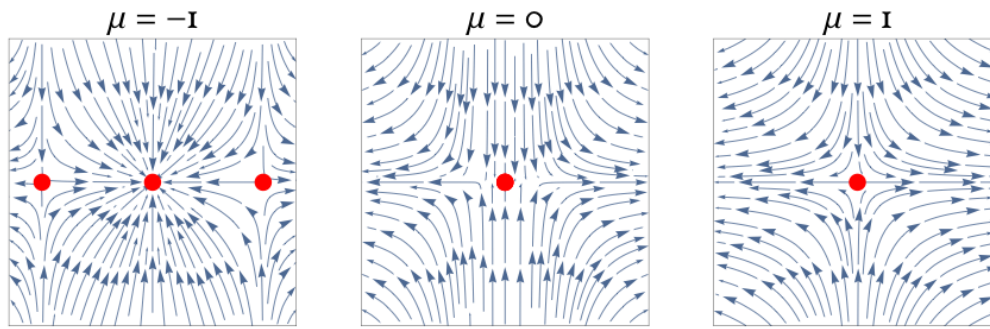
$$x, y > 0$$

in this system, bifurcation occurs on this curve

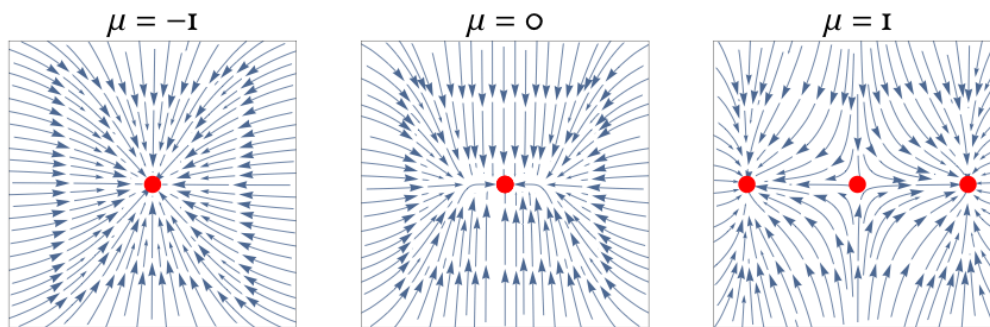


[tinyurl.com/E91bifurcations2d](http://tinyurl.com/E91bifurcations2d)

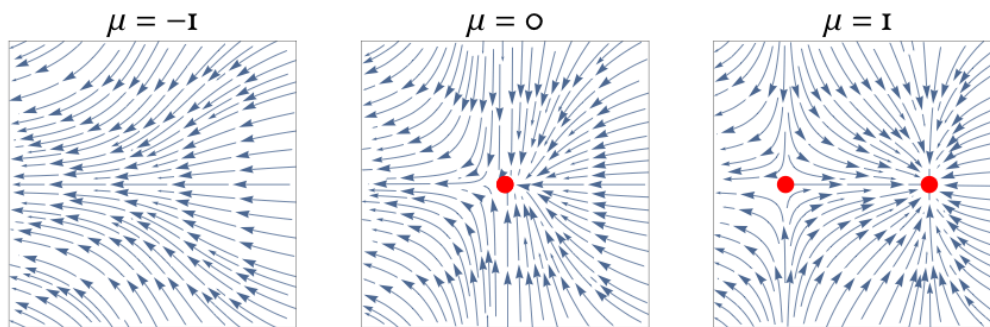
**Subcritical Pitchfork bifurcation,  $\dot{x} = \mu x + x^3, \dot{y} = -y$**



**Supercritical Pitchfork bifurcation,  $\dot{x} = \mu x - x^3, \dot{y} = -y$**



**Saddle-Node Bifurcation,  $\dot{x} = \mu - x^2, \dot{y} = -y$**



**Transcritical Bifurcation,  $\dot{x} = \mu x - x^2, \dot{y} = -y$**

