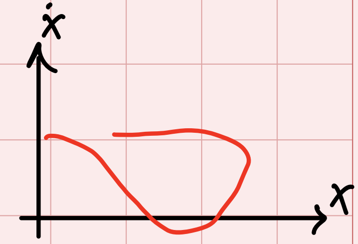
A diagram showing
- all qualitatively different trajectories
- all fixed points

Note: $f(x)$ must be a function

in addition, we will work with $[f(x)]$'s that are "nice" = sufficiently smooth.



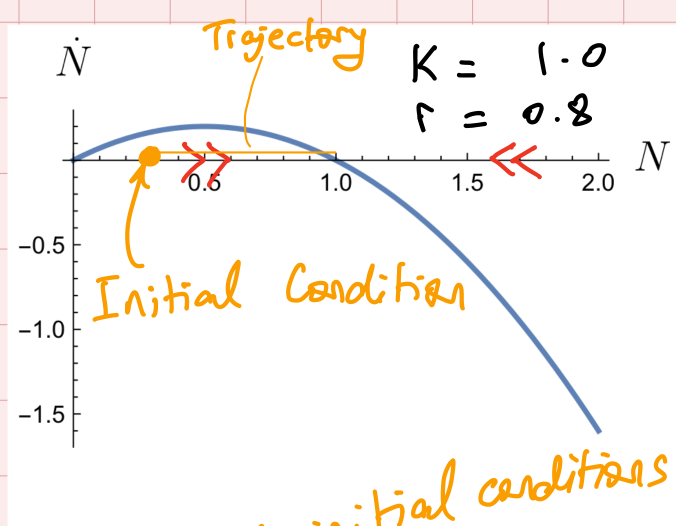
not allowed

Wed, Jan 29 Lecture 3

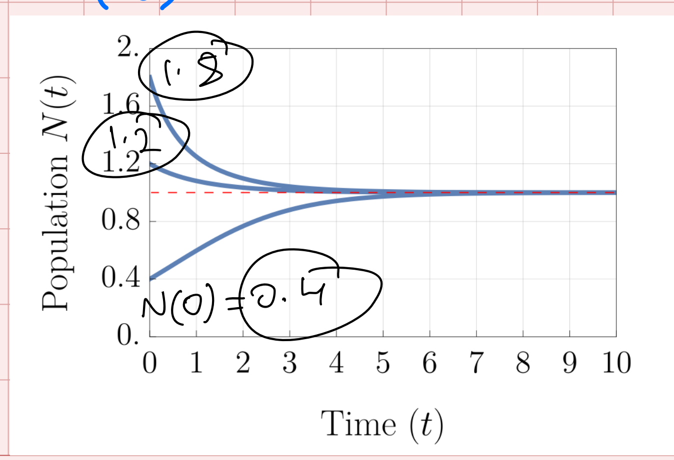
Example
$$\dot{N} = rN\left[1 - \frac{N}{K}\right]$$
Logistic Equation

carrying capacity

$[N]$ = people
$[r]$ = day^{-1}
$[K]$ = people
$[\dot{N}]$ = people/day



N(t)

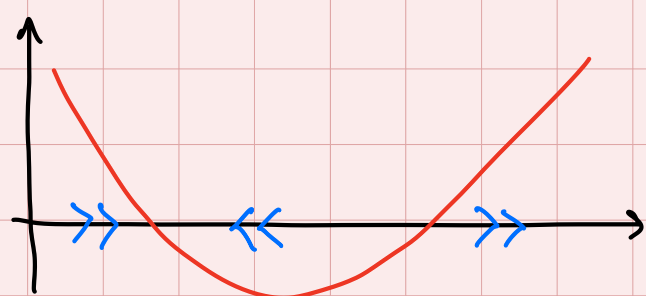


Calculating curves $N(t)$:

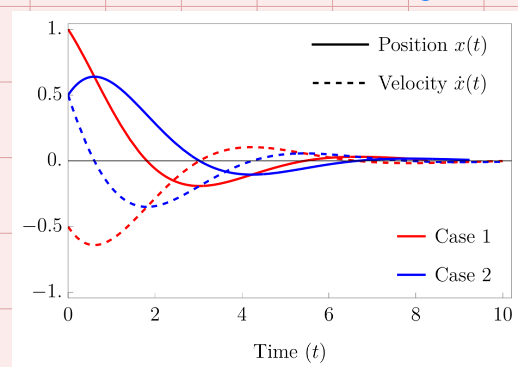
$$\frac{dx}{dt} = x(1-x) \qquad \text{with } x(0) = x_0 \text{ or, } \tfrac{1}{2}$$

<u>Note</u> : It can be shown that oscillations and "overshoot", or other <u>nonmonotonic behaviour</u> is impossible in

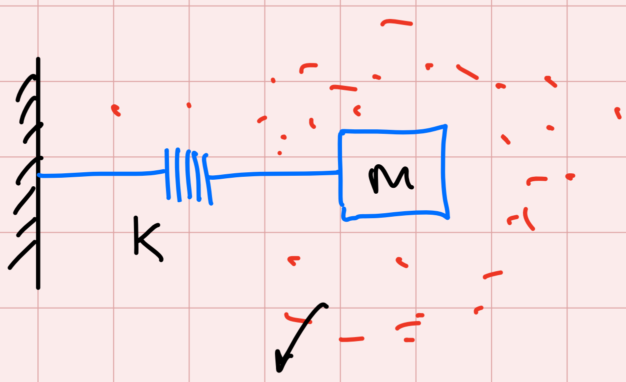
$$\dot{x} = f(x), \quad x \in \mathbb{R}^1$$



e.g.



in S.H.O.



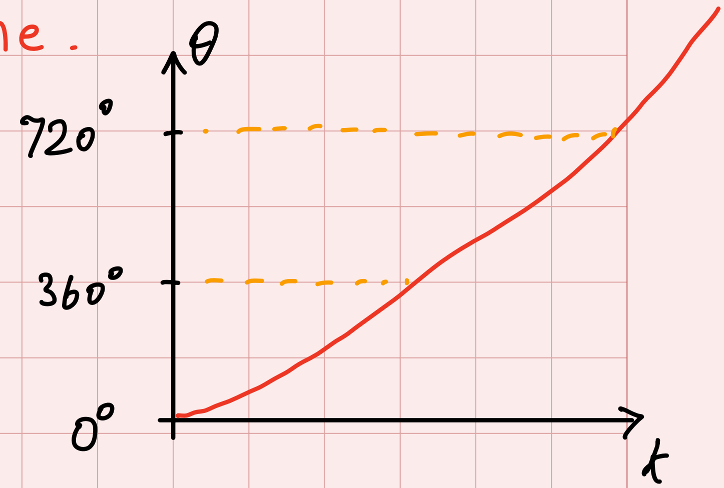
$$\underbrace{m\ddot{x}}_{\text{drop}} + c\dot{x} + kx = 0$$

very viscous fluid, c

But : $\dot{\theta} = f(\theta)$, $\theta \in [0, 2\pi)$ can have "oscillations" we reinterpret $\theta = 370^\circ$ to mean

$$\theta = 10^\circ.$$

i.e. θ lives on the circle here, not on the real line.



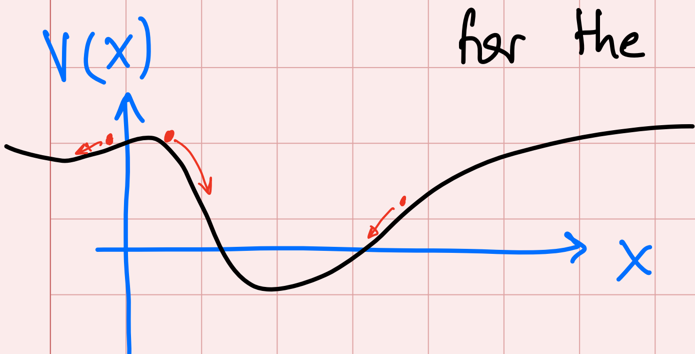
Potentials

if $\dot{x} = f(x)$ and $f(x)$ can be expressed as

$$f(x) = -\frac{d}{dx}V(x)$$

for some function $V(x)$

then $V(x)$ is called a "potential" for the dynamical system $\dot{x} = f(x)$

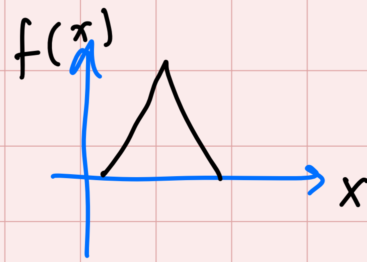


Flow occurs "downhill" in V.

Strogatz (2.7) shows that $\frac{dV}{dt} \leq 0$ along trajectories $x(t)$.

$$f(x) = 2x \longrightarrow V(x) = -x^2$$

$f(x) =$



Linear Stability Analysis of fixed points.

Suppose x^* is a value of x where $f(x^*)=0$
What happens to x if it is initialized close to x^*?

Let $\eta(t) \equiv x(t) - x^*$

$$\dot\eta(t) = \underbrace{\dot x(t)}_{f(x)} - 0$$

$$\dot\eta = \dot x = f(x)$$

$$\dot\eta = f(x) = f(x^* + \eta)$$

use Taylor series assuming η small.

$$f(x^*+\eta) = \cancelto{0}{f(x^*)} + \eta f'(x^*) + \underbrace{\frac{\eta^2}{2!} f''(x^*) + \cdots}_{O(\eta^2)}$$

$$\boxed{\dot\eta = \eta \underbrace{f'(x^*)}_{\text{const. number}} + O(\eta^2)}$$

Evolution equation for small perturbations η away from x^*.