Evolution equation for small perturbations η away from x^*.

Mon, Feb 3 Lecture 4

Bifurcations

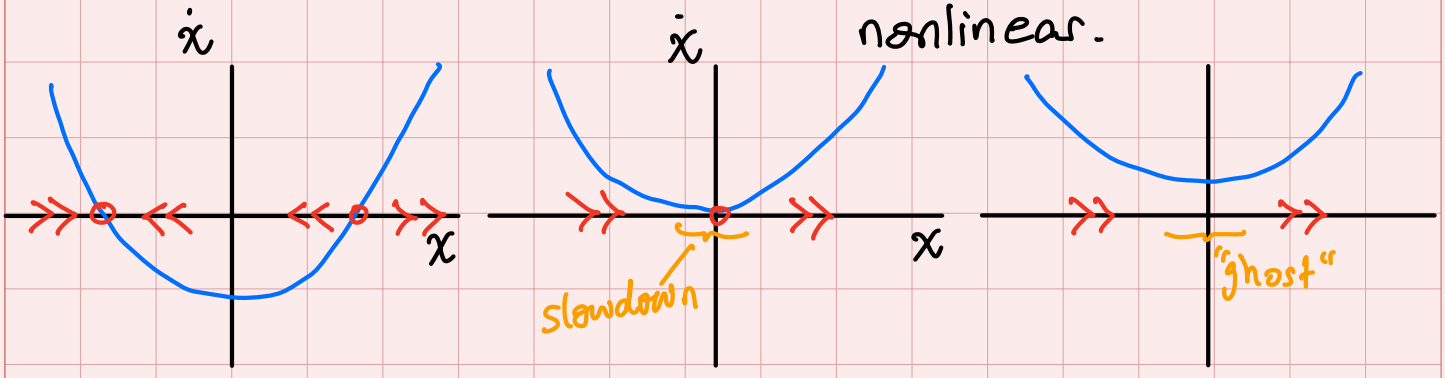
$\dot{x} = f(x; r)$ — x: independent variable; r: parameter.

Let the system be parameterized by one or more parameters 'r'. How does the qualitative behaviour of the system change with r?

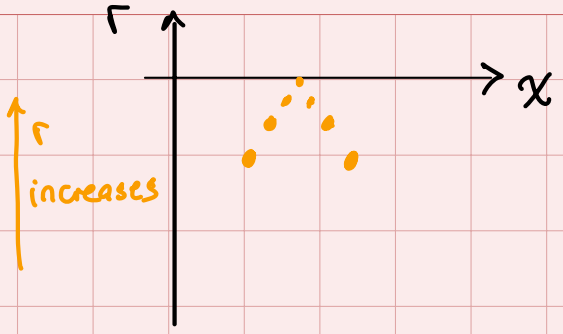
① Saddle-Node Bifurcation

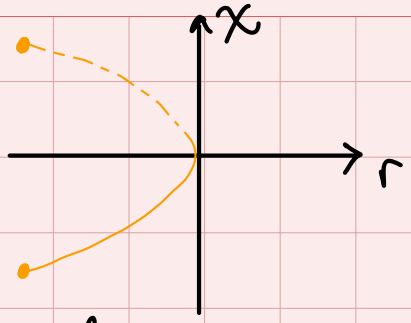
$$\dot{x} = r + x^2$$

one-dimensional first-order nonlinear.



$r < 0$ $r = 0$ $r > 0$

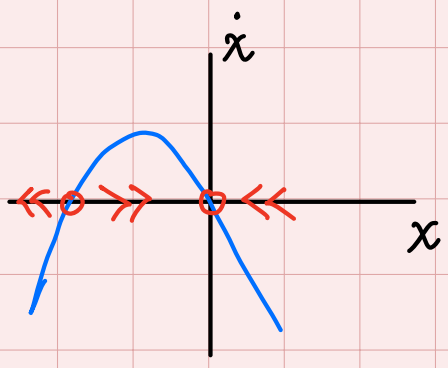




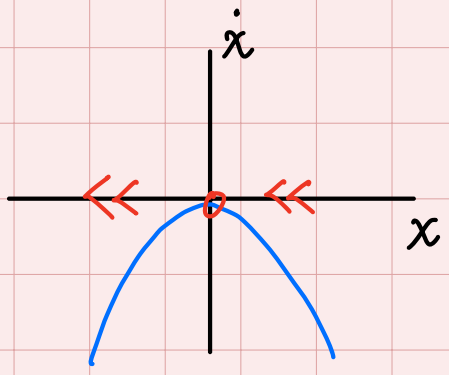
Bifurcation Diagram for Saddle-Node Bifurcation

(2) Transcritical Bifurcation

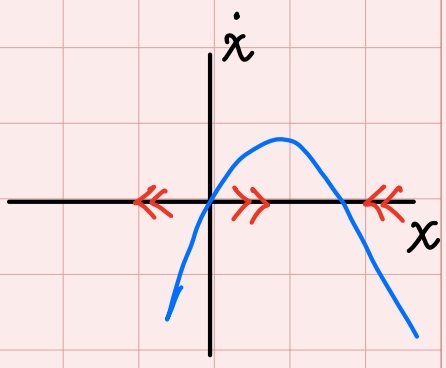
$$\dot{x} = rx - x^2$$



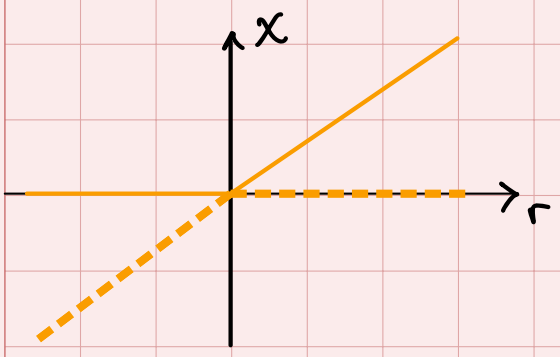
$r < 0$



$r = 0$



$r > 0$



"exchange of stability"

Bifurcation Diagram for Transcritical Bifurcation

How to calculate Bifurcation Curves

$$\dot{x} = -x + r \tanh x \qquad (A)$$

Find fixed points x^*, for which $f(x^*) = 0$

Here, f also has a parameter r.

Solve $\boxed{f(x^*; r) = 0}$ for many r's.

Gives $\{x^*, r\}$ pairs. Plot them.

with root-finding program or by hand.

e.g. for system (A)

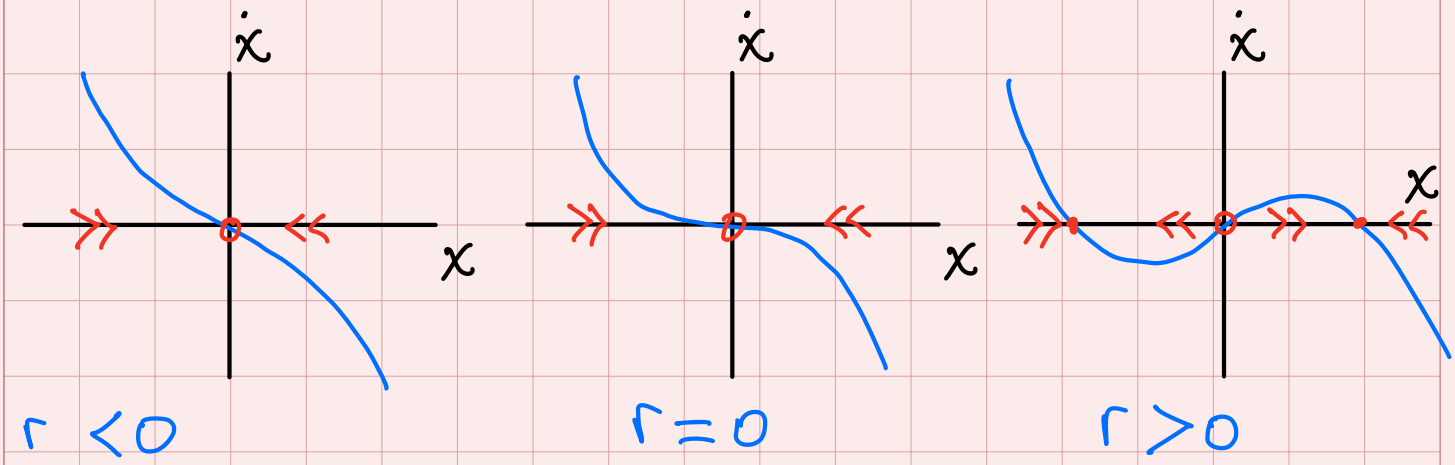
Solve $0 = -x^* + r \tanh x^*$ for x^* after setting r to some value.

Set x^* to some value, find $r = \dfrac{x^*}{\tanh x^*}$

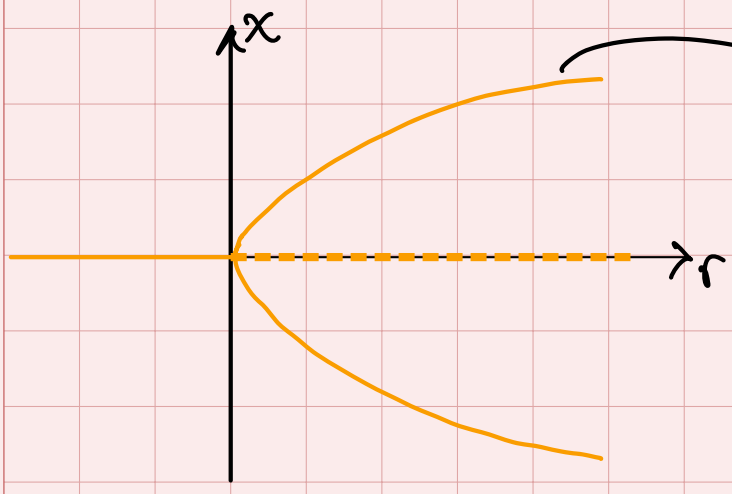
③ Pitchfork Bifurcation

$$\dot{x} = rx - x^3 \quad \text{supercritical}$$
$$\dot{x} = rx + x^3 \quad \text{subcritical}$$

→ supercritical : new fixed pts. appear above critical r.



$r < 0$ $r = 0$ $r > 0$



$x^*(r) = \ldots ? \sqrt{r} \ldots$