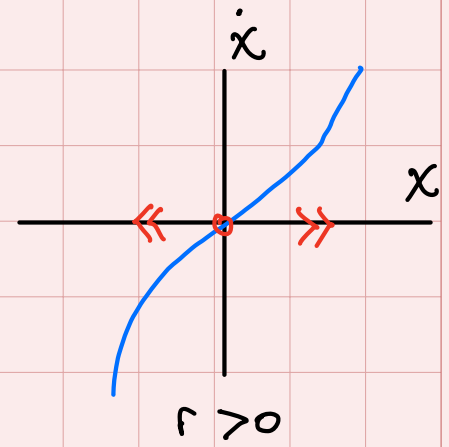**Wed, Feb 5　　Lecture 5**

→ subcritical　pitchfork　　　$\dot{x} = rx + x^3$

$r < 0$　　　　　　$r = 0$　　　　　　$r > 0$

unstable

stable　　　unstable
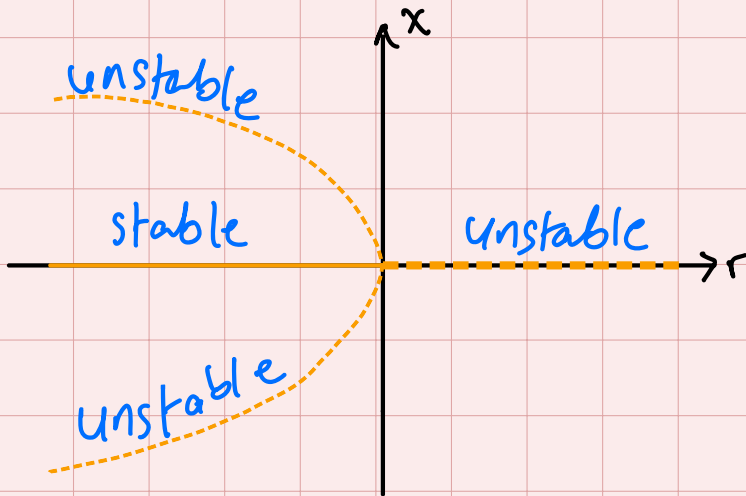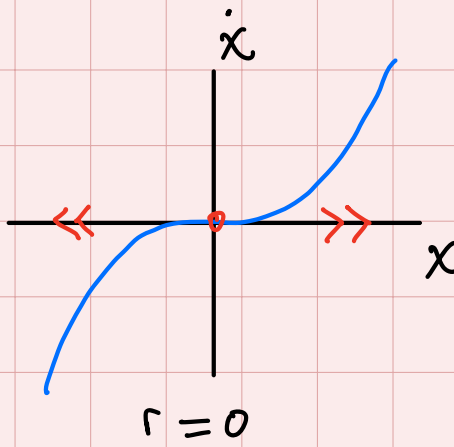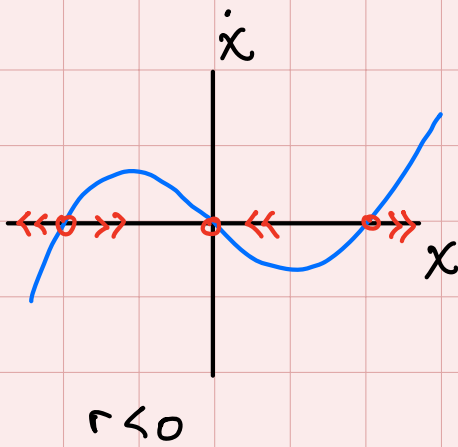
unstable

$\dot{x} = f(x;r)$	$r < 0$	$r > 0$
Saddle-node $\dot{x} = r + x^2$	1 stable 1 unstable	None None
Transcritical $\dot{x} = rx - x^2$	1 stable $\longrightarrow$ 1 unstable $\longrightarrow$	1 unstable 1 stable
Pitchfork:		
Supercrit. $\dot{x} = rx - x^3$	None $\longrightarrow$ 1 stable $\longrightarrow$ None $\longrightarrow$	1 stable 1 unstable 1 stable
subcrit. $\dot{x} = rx + x^3$	1 unstable $\longrightarrow$ 1 stable $\longrightarrow$ 1 unstable $\longrightarrow$	None 1 unstable None

In practice, systems
with a subcritical
pitchfork bifurcation
don't actually go off to $\pm\infty$
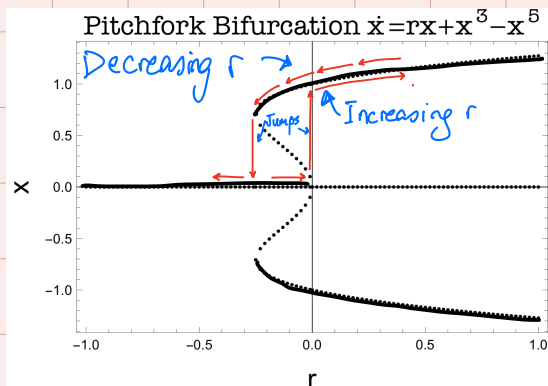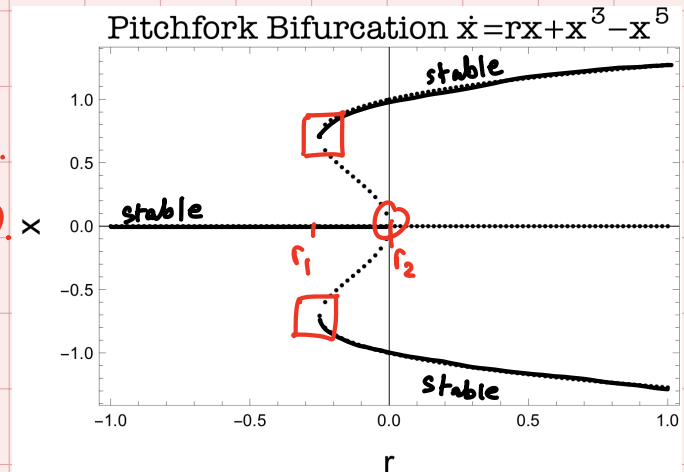instead, higher-order terms
play a stabilizing role.

$$\dot{x} = rx + x^3 - x^5$$

unstable

stable   unstable

unstable
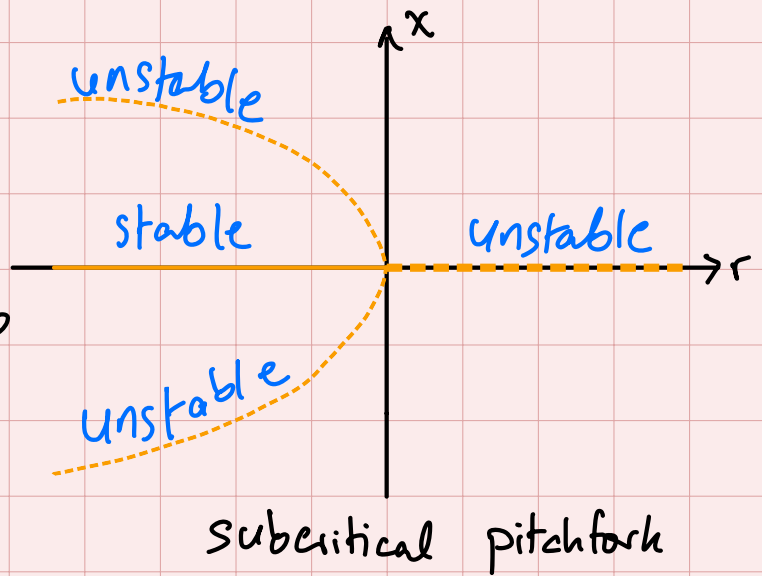
Subcritical pitchfork

for small x, this governs the dynamics.
for large x, the x^5 term plays a role also.

→ Saddle-node bifurcation at □.

→ Subcritical pitchfork bif. at ○.

Pitchfork Bifurcation $\dot{x} = rx + x^3 - x^5$

stable

stable

stable

r_1 r_2

stable

Pitchfork Bifurcation $\dot{x} = rx + x^3 - x^5$

Decreasing r

Jumps   Increasing r

Hysteresis